ACTIVE SUSPENSION: FUTURE LESSONS FROM THE PAST

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ABSTRACT

Active suspension was a topic of great research interest near the end of the last century. Ultimately broad bandwidth active systems were found to be too expensive in terms of both energy and financial cost. This past work, developing the ultimate vehicle suspension, has relevance for today's vehicle designers working on more efficient and effective suspension systems for practical vehicles. From a control theorist's perspective, it provides an interesting case study in the use of "practical" knowledge to allow "better" performance than predicted by theoretically optimal linear controllers.

A brief history of active suspension and the specific founding contributions of William F. Milliken will be introduced. Milliken collaborated with Peter Wright, David Williams, and others at Lotus to further develop their Lotus modal control concept. In a parallel effort, Dean Karnopp and Don Margolis presented the notion of inertial (Skyhook) damping. These concepts will be compared, the combination of these efforts will be discussed, and eventual vehicle results presented.

Most of the contemporary literature treated active suspension as a theoretical vibration isolation problem, but handling improvements from active suspension were even more impressive. Handling and actual hardware considerations motivated the confluence of both primary approaches. This innovative implementation of a control algorithm preserving features of both Lotus modal control and inertial damping is discussed, and compared with theoretical optimal controllers. Finally, a surprising fundamental performance limit of the modal inertial damping algorithm is discussed, and a solution presented.

INTRODUCTION

Suspensions transmit forces from the suspended or sprung mass of the vehicle to the road, through the tires. Historically such suspensions were comprised of springs and dampers. These elements are said to be "passive" from the standpoint that they either dissipate or store energy that ultimately comes from the vehicle traveling over a rough curvy road. The suspension serves several purposes: 1) to locate the sprung and unsprung masses, 2) to isolate the sprung mass from vertical vibrations, 3) to transmit horizontal forces that serve to accelerate, brake, and turn the vehicle. These sometime conflicting goals, as well as practical constraints, have led to a variety of suspension configurations. Passive suspensions have long been an acceptable compromise of these goals as espoused by the inaugural Milliken Lecturer in his popular vehicle dynamics text (1).

In active suspension, components are added that have the capability to add energy to the suspension. To support the added cost and complexity of these systems, they reduce compromises inherent in passive suspensions. For example, for good vertical vibration isolation the spring elements should be soft. But to positively locate the sprung mass when turning or braking, they should be stiff. Ideally dampers should absorb energy at resonant frequencies. Vertical forces transmitted through the suspension should act in concert with the horizontal forces required to maneuver the vehicle. A given corner of the vehicle should isolate the sprung mass from vertical road disturbances while supporting vehicle dynamic induced inertia loadings (pitch and roll) without deflection, and provide the optimal vertical load through the tire's contact patch. Passive suspensions have been doing most of these things most of the time in a very cost effective manner (2).

In a typical embodiment, a simple hydraulic damper that dissipates energy might be replaced by a hydraulic actuator: a cylinder whose internal pressure is controlled by an electrically actuated servovalve. Because the servovalve is electrically actuated, it can be computer controlled. Active suspension is one application of a general movement in the latter part of the 20th century to apply the newly emerged control theory body of knowledge to ground vehicles, and more particularly to chassis control systems. Before control theory could be applied, the ground vehicle had to be viewed as a dynamic system. Bill Milliken applied expertise gained in dynamically characterizing
World War II aircraft to post-war passenger cars. Later, a member of his team wrote the dynamic equations of motion - the mathematical characterization of the relevant properties of a ground vehicle to control (3).

Lotus was well known for pushing boundaries in automotive engineering. Below highway speeds aerodynamics are a negligible external force on the vehicle. At racing speeds, they are critical. Under the technical direction of Peter Wright, who began his career as an aerodynamicist, Lotus began to look to computer controlled suspension for a competitive advantage. In 1979 Lotus asked David Williams, who did flight control on the British Harrier military aircraft to do the algorithm work at the Cranfield Institute of Technology, as shown in Figure 1. Bill Milliken was a consultant to Lotus but his contributions went beyond the technical aspects. Lotus began to experiment with active suspension using electro-hydraulic actuation. Bill Milliken had long known Bill Moog, the entrepreneurial founder of the upstate New York servovalve company that bears his name. If Dave Williams’ controller was the brain of the early Lotus active suspension, the Moog servovalve was its heart. Bill Milliken introduced Moog and Lotus, forming a joint venture company called Active Control Systems or ACS, in Stuart, Florida where a small effective team of highly competent engineers built on the fundamental work started by Bill Milliken, Peter Wright, and David Williams (4, 5).

What follows in this work is a description of the development of active suspension from the “Lotus” perspective, which is to say of someone who learned the principles of active suspension from the Lotus organization. The Lotus system was “broad bandwidth” meaning that the hydraulic actuation was fast enough to lift the wheel up over a bump and set it down on the other side. This is in distinction to low bandwidth systems, that only use energy to react to the lower frequency inertia loadings on the sprung mass, and rely on passive elements for vertical vibration isolation.

In the 1980s there was an incredible amount of research on active suspension. Using control theory many of the compromises wrestled with by vehicle designers were eliminated. Lotus was not the only organization interested in active suspension. Independently, Professors Dean Karnopp and Don Margolis at UC Davis developed the simple and yet powerful concept of inertial damping, and coined the phrase ‘skyhook damping.’ The skyhook damping concept will be discussed, and then the traditional Lotus modal control will be presented. The Lotus algorithm will be seen to be more complicated than inertial damping, but also allowing for a better handling vehicle. The independent concepts of Lotus modal control and skyhook damping were combined at ACS. Actual vehicle results show this combination performing significantly better in terms of ride quality than the preceding Lotus modal control. Finally this work closes with a solution to the fundamental performance limit of the combined system.

It will be shown that the Lotus modal control algorithm was formulated for a specific hardware configuration. The skyhook damper was theoretically formulated for an unrealizable hardware configuration, but can be approximated and practically implemented by a broad class of hardware. The combined modal inertial damping can similarly be implemented by a variety of hardware configurations.

It is rare that an engineer gets to develop the ultimate performing system. In the end, despite the impressive functional improvements in both ride and handling described herein, the Lotus hardware configuration - broad bandwidth active suspension - was too expensive in terms of both financial cost and energy use. Important lessons can be drawn from the effort, however. The result was a vehicle providing unprecedented ride quality and perfect neutral steer, and serves to define the performance envelope of a ground vehicle. A case study can also be made of how such performance was possible. It was a blend of control theory and vehicle dynamics, of theory and practice, of linear and nonlinear systems. It started as blend of Karnopp's skyhook damper and the Lotus modal control.
**INERTIAL DAMPING**

George E.P. Box once famously quipped “all models are wrong, but some are useful.” The quarter-car model shown Figure 2 embodies this utility capturing many of the important vibration isolation properties that characterize a passive suspension. The passive elements could be nonlinear, they are essentially energy dissipation and storage elements between discrete masses. In the most common uses of the quarter-car model, these elements are linear, so that it is possible to speak of two natural frequencies. If one considers the sprung mass so large as to make the unsprung mass negligible, the sprung mass is supported by the series combination of the suspension and tire springs. This frequency is usually around 2 Hz, and is termed the ride frequency. Another natural frequency is found when the much heavier sprung mass is considered practically infinite, and the smaller unsprung mass is located by the parallel combination of the suspension and tire springs. This frequency is called the wheel hop frequency and is usually between 10-12 Hz.

The tire is usually very lightly damped, so most of the required damping in quarter-car model is in the suspension. Passengers are located on the sprung mass, and are sensitive to its movement. Without this damping the sprung mass is undamped at the ride frequency and thus offensive to occupants. Unfortunately the damper creates a force based on relative motion between the sprung and unsprung masses, transmitting forces to the sprung mass. So in the case of relative damping, there is a trade-off between damping the sprung mass resonance, and transmitting forces at other frequencies to the occupant cabin.

Figure 3 shows an idealized situation where the sprung mass damper is inertially grounded. In this physically unrealizable case unprung mass motion does not create a force on the sprung mass through the damper. There is no trade-off between sprung and unsprung masses, and inertial damping gains can be quite high. Dean Karnopp was the first to realize this, and coined the phrase “sky-hook” damping. In practice this system is implemented by a force actuator driven by a sprung mass velocity signal, and the force actuator generates a reaction force on the unsprung mass.

From a control systems perspective, inertial damping can be thought as a closed-loop on inertial velocity with a zero set point as shown in Figure 4. From classical control theory we appreciate the closed-loop stability properties found in the examination of the open-loop. The sprung mass resonance is clearly seen in the magnitude part of the bode plot shown in Figure 5, which is the open-loop frequency response of the inertially damped system of Figure 4. What is more insightful is the high frequency phase lag. The inertial damping open-loop does not reach the 180 degrees necessary for closed-loop instability. Inertial damping is in a class of dynamic systems referred to formally as positive real - characterized by their stability (6,7 and 8). So inertial damping is a very powerful concept with two very important attributes that will be revisited several times throughout this work: 1.) because there is no compromise in the damping, desired inertial damping levels are quite high, and 2.) because of the positive real stability properties of the skyhook damping configuration, the ideal inertial damping level is not bounded by system stability. Thus high inertial damping is both desireable and realizable.

When the inertial damping principle is used in practice, the actuator will have a bandwidth. Actuator dynamics will add
phase lag, and at some point the open-loop response will cross the 180 degree point and the inertial damping gain will be limited by stability as shown in Figure 6. Fortunately it is possible to find such actuators with sufficient bandwidth, and Karnopp's skyhook damper provides a very convenient tool to quickly characterize their 1/4 car model performance.

Mehdi Ahmadian suggested "groundhook damping," of the unsprung mass (9). Rather than drive the force actuator with the sprung mass inertial velocity, it is possible to drive the force actuator with unsprung mass inertial velocity. This has the advantage of damping the hub to provide a more constant friction circle to transmit horizontal forces through the contact patch of the tire. Of course the actuator generates its hub damping force by pushing against the sprung mass, degrading ride. In practice it would be possible to transition from Karnopp's skyhook to Ahmadian's groundhook based on lateral acceleration, degrading the ride only when handling is a priority.

Davor Hrovat, a graduate student of Prof. Karnopp, showed that such a skyhook damper placed at each corner of a sprung mass, will produce total vehicle vibration isolation results nearly equal to a full state optimal controller (10).

\[
\begin{bmatrix}
  F_{1a} \\
  F_{2a} \\
  F_{3a} \\
  F_{4a}
\end{bmatrix} =
\begin{bmatrix}
  C_{d_{1a}} & 0 & 0 & 0 \\
  0 & C_{d_{2a}} & 0 & 0 \\
  0 & 0 & C_{d_{3a}} & 0 \\
  0 & 0 & 0 & C_{d_{4a}}
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_{1a} \\
  \dot{x}_{2a} \\
  \dot{x}_{3a} \\
  \dot{x}_{4a}
\end{bmatrix}
\]

Equation 1

where \( F_{ia} \) is the first corner force, \( \dot{x}_{ia} \) is the first corner velocity and \( C_{des} \) is the desired damping.

So inertial damping provides a readily understandable 1/4 car solution that can be scaled up to the complete vehicle. Such a vehicle has some shortcomings. It has a "magic carpet" ride, that is somewhat disconnected from the road. To provide this magic carpet ride, the suspension travel must be large enough to accommodate bumps, even hills. At extreme levels the skyhook damper will attempt to keep the inertial velocity zero while the vehicle is climbing a hill, and eventually the suspension will run out of travel. If the vehicle is allowed to slowly recover its ride height it will be perceived as disconnected, and if it recovers too fast it will be transmitting discomfiting accelerations to the occupants in the sprung mass. Furthermore, these disconnected inputs can send the wrong cues to the driver in a handling situation where pitch and roll moments are produced. All of these practical issues aside, inertial damping is a very powerful concept for vibration isolation and will be revisited.

**LOTUS MODAL CONTROL**

The LOTUS algorithm can also be developed for a quarter-car model and compared with the quarter-car "skyhook" inertial damping algorithm. In its simplest practical implementation shown in Figure 9, force through the suspension is measured by a load cell. Unsprung mass acceleration is measured by an accelerometer and integrated to yield hub velocity. A hydraulic cylinder replaces the damper in parallel with the load spring that supports the static weight of the vehicle. Inside the cylinder is a position sensing device, and flow in and out of the cylinder is controlled by a servovalve. Second order dynamics can be assigned to the sprung mass,

\[
m_s \ddot{z}_s + C_{des} \dot{z}_s + K_{des} z_s = 0
\]

Equation 2
where $C_{\text{des}}$ and $K_{\text{des}}$ are the desired damping and stiffness respectively, $z_s$ is the displacement of sprung mass $m_s$. It is important to note that at this point $C_{\text{des}}$ is exactly equivalent to the skyhook damping gain from the previous section. The way the suspension model is constructed, all force to the sprung mass is transmitted through the load cell, $F_{\text{load cell}}$ although in practice that is not a requirement (11).

$$F_{\text{load cell}} = -m_s \ddot{z}_s$$

Equation 3

Rather than multiplying the stiffness by the sprung mass displacement, it is multiplied by the measured relative displacement. The desired stiffness $K_{\text{des}}$ thus acts relative to the hub displacement, and is not inertially referenced. Practically speaking, the fact that this stiffness is relative will ensure proper location of the sprung mass relative to the undulating road surface.

Finally, the sprung mass velocity can be expressed in terms of hub velocity and strut relative velocity. Hub velocity is available as integrated hub acceleration. So actuator velocity $\dot{z}_a$ is the difference between unsprung mass velocity $\dot{z}_u$ and sprung mass velocity $\dot{z}_s$.

$$\dot{z}_a = \dot{z}_u - \dot{z}_s$$

Equation 4

Inserting Equation 3 and Equation 4 into Equation 2, the relative strut velocity can be solved for in terms of measured variables.

$$\dot{z}_a = -\frac{1}{C_{\text{des}}} F_{\text{load cell}} + \dot{z}_u - \frac{K_{\text{des}}}{C_{\text{des}}} (z_a)$$

Equation 5

Equation 5 expresses strut velocity as a function of measured (or in the case of hub velocity computed) variables. Strut velocity is determined by flow through the controlling servovalve.

The "heart" of the Lotus active suspension system is the Moog servovalve, and its operational characteristics in large
measure determined the structure of the Lotus modal control algorithm. The Lotus control strategy assumed the servovalve produced the exact required flow in an open-loop sense. This was not a bad assumption. The Moog valve is a true marvel of mid-20th century engineering. Its linearity and hysteresis shown in Figure 8, and its bandwidth shown in Figure 9 were amazing for a mechanical system, owing to its two-stage concept with mechanical closed-loop coupling between stages.

The quarter-car version of the Lotus modal control assumes the second-order dynamics of Equation 2 and uses measured (and calculated) parameters to produce the open-loop valve drive of Equation 5. It is possible to apply this quarter-car model to a complete vehicle assuming the sprung mass is supported at each corner by the same quarter-car model.

Just as Equation 2 assumes second-order dynamics for the single vertical sprung mass degree of freedom in the quarter-car model, it can be used for the vertical degree of freedom of the full vehicle sprung mass.

\[ m \ddot{z} + C_{\text{heave}} \dot{z} + K_{\text{heave}} z = 0 \]

**Equation 6**

where \( z \) is the heave mode displacement; \( C_{\text{heave}} \) and \( K_{\text{heave}} \) are the desired heave mode damping and stiffness respectively.

Similar second order dynamic equations can be written for the pitch and roll sprung mass modes,

\[ I_x \ddot{\phi} + C_{\text{roll}} \dot{\phi} + K_{\text{roll}} \phi = 0 \]

**Equation 7**

where \( \phi \) is the roll mode displacement, \( I_x \) the roll moment of inertia, and \( C_{\text{roll}} \) and \( K_{\text{roll}} \) the desired roll damping and stiffness, and

\[ I_y \ddot{\theta} + C_{\text{pitch}} \dot{\theta} + K_{\text{pitch}} \theta = 0 \]

**Equation 8**

where \( \theta \) is the pitch mode displacement, \( I_y \) is pitch moment of inertia, and \( C_{\text{pitch}} \) and \( K_{\text{pitch}} \) are the desired pitch damping and stiffness.

A key to understanding the Lotus control of the quarter-car model is how assumed second order dynamics resulted in an open-loop valve drive. Expanding the application to the full vehicle, a second key principle is required. Three rigid body modes are evident in Figure 10: heave, pitch and roll. A fourth mode is added, warp.

\[ I_{\text{warp}} \ddot{\psi} + C_{\text{warp}} \dot{\psi} + K_{\text{warp}} \psi = 0 \]

**Equation 9**

where \( \psi \) is warp displacement, \( I_{\text{warp}} \) is the warp moment of inertia, and \( C_{\text{warp}} \) and \( K_{\text{warp}} \) are the desired warp damping and stiffness.

Warp can superficially be thought of as a torsional or twisting mode of the sprung mass. This is not actually the case, as the sprung mass is assumed rigid, and therefore there can be no inertially measured warp displacement. As will eventually be shown the warp mode achieves two related goals. First, the addition of the warp mode allows an invertible transformation between corner displacements and modal displacements. Second, the warp mode allows the Lotus algorithm to affect vehicle handling via dynamic weight distribution.

Equation 6 through Equation 9 are written in modal coordinates, but the same actual sensors from the quarter-car model are used at each corner.

\[ z_1 = z - \frac{t}{2} \phi - a \theta - \frac{1}{2} \psi \]

**Equation 10**

\[ z_2 = z + \frac{t}{2} \phi - a \theta + \frac{1}{2} \psi \]

**Equation 11**

\[ z_3 = z + \frac{t}{2} \phi + b \theta - \frac{1}{2} \psi \]

**Equation 12**

\[ z_4 = z - \frac{t}{2} \phi + b \theta + \frac{1}{2} \psi \]

**Equation 13**

where \( z_i \) is displacement of the \( i \)th corner, \( t \) is the vehicle track width, \( a \) and \( b \) are the distances from the center of mass to the front and rear axles respectively (12).

Therefore a transformation \( T \) is required to convert modal coordinates to corner coordinates,

\[ x_{\text{corner}} = T x_{\text{modal}} \]

**Equation 14**

where,

\[ x_{\text{corner}} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \]

**Equation 15**

\[ x_{\text{modal}} = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \end{bmatrix} \]

**Equation 16**
Similarly forces measured at each corner can be combined to form modal forces,

\[ F_{\text{heave}} = F_1 + F_2 + F_3 + F_4 \]

\[ M_{\text{roll}} = \frac{t}{2}(-F_1 + F_2 + F_3 - F_4) \]

\[ M_{\text{pitch}} = -a(F_1 + F_2) + b(F_3 + F_4) \]

\[ M_{\text{warp}} = \frac{t}{2}(-F_1 + F_2 - F_3 + F_4) \]

Equation 18

Equation 19

Equation 20

Equation 21

where \( F_i \) is the force measured on the load cell at the \( i \)th corner. These modal forces are assumed to produce modal accelerations, similar to Equation 3 for the quarter-car heave mode.

Equation 18 through Equation 21 express the equivalent of the measured load cells at each corner of the quarter-car model. These equations can be written in a very convenient form with useful similarity to Equation 14,

\[ F_{\text{modal}} = T^T F_{\text{corner}} \]

Equation 22

where

\[ F_{\text{corner}} = \begin{bmatrix} F_{1a} \\ F_{2a} \\ F_{3a} \\ F_{4a} \end{bmatrix} \]

\[ F_{\text{modal}} = \begin{bmatrix} F_{\text{heave}} \\ M_{\text{roll}} \\ M_{\text{pitch}} \\ M_{\text{warp}} \end{bmatrix} \]

Equation 23

Equation 24

and \( T \) is the same as Equation 14 and given by Equation 17.

From them, load cell measurements are combined at each corner and combined to form the modal products of mass and acceleration terms in Equation 6 through Equation 9. To apply the Lotus modal control strategy to the full vehicle, we must find a modal version of Equation 5 that expresses strut velocities at each corner in terms of desired modal stiffness and dampsings, as well as measurements made at each corner. Equation 14 and Equation 22 are inserted into Equation 5.

\[ \ddot{z}_a = T C_{\text{modal}}^{-1} T^T F_{\text{corner}} - T C_{\text{modal}}^{-1} K_{\text{modal}} T^{-1} \dot{z}_a + \dot{u} \]

Equation 25

where

\[ C_{\text{modal}} = \begin{bmatrix} C_{\text{heave}} & 0 & 0 & 0 \\ 0 & C_{\text{roll}} & 0 & 0 \\ 0 & 0 & C_{\text{pitch}} & 0 \\ 0 & 0 & 0 & C_{\text{warp}} \end{bmatrix} \]

Equation 26

\[ K_{\text{modal}} = \begin{bmatrix} K_{\text{heave}} & 0 & 0 & 0 \\ 0 & K_{\text{roll}} & 0 & 0 \\ 0 & 0 & K_{\text{pitch}} & 0 \\ 0 & 0 & 0 & K_{\text{warp}} \end{bmatrix} \]

Equation 27

Equation 26 and Equation 27 are modal dampings and stiffness that are subjectively tuned for good vehicle ride. Given these values, measured loads and actuator displacements, and calculated hub velocities at each corner, actuator velocities, and therefore open-loop valve drive signals, are created for each corner.

From Equation 25 it is evident that the coordinate transformation of Equation 17 must be invertible, therefore requiring the warp mode or some other linearly independent mapping of corner displacements into a modal coordinate. This particular modal coordinate, the warp mode, was chosen with care, however.

The roll moment at the front and rear axles can be independently expressed.

\[ M_{\text{front}} = \frac{t}{2}(F_1 - F_2) \]

Equation 28

\[ M_{\text{rear}} = \frac{t}{2}(F_4 - F_3) \]

Equation 29

The roll and warp modes previously defined in Equation 19 and Equation 21 can be expressed in terms of Equation 28 and Equation 29.
\[ M_{roll} = M_{front} + M_{rear} \]

Equation 30

\[ M_{warp} = M_{front} - M_{rear} \]

Equation 31

Dividing Equation 31 by Equation 30 yields a roll moment distribution \( \varepsilon \). If all the roll moment is on the front \( \varepsilon = 1 \), if all on the rear \( \varepsilon = -1 \), and if balanced \( \varepsilon = 0 \). It is well known in vehicle handling that if the roll moment is more resisted by the front suspension the vehicle will tend toward understeer, and at the rear, tend toward oversteer (13).

\[ \varepsilon = \frac{M_{front} - M_{rear}}{M_{front} + M_{rear}} \]

Equation 32

The roll moment distribution parameter is simply the warp mode moment divided by the roll mode moment. Specifically, the warp moment determines the sign of the roll moment distribution, and therefore if the vehicle will be more understeering or oversteering.

\[ \varepsilon = \frac{M_{warp}}{M_{roll}} \]

Equation 33

The problem with Equation 33 is that the roll moment distribution \( \varepsilon \) is not directly selectable. Rather, in view of Equation 7 and Equation 9, understanding that the modal roll and warp moments are constructed from load cell measurements at each corner, Equation 33 can be rewritten in terms of modal states and desired modal dynamic characteristics.

\[ \varepsilon = \frac{C_{warp}\dot{\psi} + K_{warp}\psi}{C_{roll}\dot{\phi} + K_{roll}\phi} \]

Equation 34

Thus the roll moment distribution is indirectly determined by the warp and roll modal states, and the respective roll and warp modal damping and stiffness parameters.

The warp mode was necessary to provide an invertible transformation between corner and modal coordinates. It has also been shown to determine the effect of active suspension on handling. The warp mode parameters of Equation 9 can thus be varied to affect the roll moment distribution of Equation 34 and thereby vehicle handling.

In steady state the time derivatives of Equation 25 go to zero, and a relationship can be found between corner forces and corner displacements.

\[ F_{corner} = T^{-1}K_{modal}T^{-1}z_a \]

Equation 35

Equation 35 can be multiplied out to form a stiffness matrix relating single wheel displacements to corner forces,

\[
\begin{bmatrix}
\frac{\partial^2}{\partial \varepsilon_1^2}
\frac{\partial^2}{\partial \varepsilon_1 \partial \varepsilon_2}
\frac{\partial^2}{\partial \varepsilon_2^2}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1
\varepsilon_2
\end{bmatrix}
\begin{bmatrix}
\frac{\partial}{\partial \varepsilon_1}
\frac{\partial}{\partial \varepsilon_2}
\end{bmatrix}
\]

Equation 36

where some nomenclature is adopted for brevity.

\[ K_h = \frac{abK_{heave}}{4(a+b)^2} \]

Equation 37

\[ K_r = \frac{K_{roll}}{4t^2} \]

Equation 38

\[ K_p = \frac{K_{pitch}}{4(a+b)^2} \]

Equation 39

\[ K_w = \frac{K_{warp}}{4t^2} \]

Equation 40

The main diagonal of the stiffness matrix of Equation 36 are single wheel stiffnesses, or in other words the force at a corner in response to a displacement of that same corner. In general, for good sprung mass behavior it is desired for the heave stiffness to be low, producing a ride frequency below peak human sensitivity, and pitch and roll stiffnesses high to minimize pitch and roll deflections in response to vehicle dynamic inertia loadings. Warp stiffness is found from handling considerations. These modal stiffnesses can be specified in this manner, and the sprung mass response to inertia loadings will be determined by the differential Equation 6 through Equation 8. However the sprung mass also receives inputs from the road. It can be seen from the diagonals of Equation 36 that as the roll, pitch and warp stiffnesses are increased, single wheel events are transmitted to the sprung mass through this increased single wheel stiffness even though the heave stiffness is low.

Through the modal transformations, minimizing roll and pitch displacements adversely affect the behavior in response to single wheel inputs. Furthermore, by modifying warp mode parameters, handling effects are coupled to single wheel effects. These issues presented trade-offs or compromises in parameter
selection. But the Lotus modal control had a more fundamental performance limit, buried in the basic single degree of freedom fundamental open-loop valve drive Equation 5.

\[
\ddot{z}_a = -\frac{1}{C_{des}} F_{load\ cell} + \dot{z}_a - \frac{K_{des}}{C_{des}} (z_a) \quad \text{(Equation 5 repeated)}
\]

Equation 5 is an ad hoc control algorithm, assuming that with sufficient variables measured or calculated and sufficient parameters selected, the resulting output would drive the actuator to achieve the desired second-order dynamics. Equation 5 was developed without consideration of closed-loop stability. In fact, when Equation 5 is implemented at least two stability issues arise.

First, the hub velocity term needs to be unity for correct behavior. Actuator dynamics are included and Equation 5 is used to drive the quarter-car model of Figure 9. The force loop gain of -1/C_{des} is set to a 6 dB stability margin, and the root locus plot shown in Figure 11 is created as the gain on hub velocity goes from zero to the value of unity required for the desired second order dynamics.

As the hub velocity gain goes from zero to unity, the system is destabilized as roots approach the right half plane.

\[\left| s \right| = \sqrt{\frac{K_{des}}{C_{des}}} \]

Since the damping is relative, force loop gains are desirably high. The limit on force loop gains is dependent upon actuator dynamics, the second stability concern associated with Lotus modal control. To summarize, when stability is taken into account, the Lotus damping is relative, and its lower limit is determined by force loop stability.

Upon further examination, we have seen the Lotus modal control to increase single wheel stiffness, to indirectly distribute the roll moment, and to provide stability-limited relative heave damping. But the Lotus modal control did allow a measure of roll moment distribution to allow the suspension to effect handling, and allow independent specification of modal dampings so that the vehicle could be soft in heave and stiff in roll and pitch.

Furthermore, the form of Equation 5 readily allows for mitigation of acknowledged compromises in suspension design. Karl Hedrick (2015 Milliken Lecturer) identified certain tradeoffs in any linear suspension design (14). Minimizing sprung mass acceleration to improve occupant comfort is a primary purpose of all suspensions. Unsprung mass deflection results in fluctuations in normal force through the contact patch of the tire, and lateral side force produced by the tire is a nonlinear function of this normal force. Therefore unsprung mass deflections result in reduced lateral force. Ideally the wheel should follow road surface deviations without variation of force through the contact patch of the tire. Practical suspensions all have a finite rattle space constraint. Sprung mass acceleration, suspension rattle space, and unsprung mass displacement all are compromised. For example, a reduction in sprung mass deflections result in reduced lateral force. Ideally the wheel should follow road surface deviations without variation of force through the contact patch of the tire. Practical suspensions all have a finite rattle space constraint. Sprung mass acceleration, suspension rattle space, and unsprung mass displacement all are compromised. For example, a reduction in sprung mass acceleration requires an increase in suspension rattle space. Similarly, minimizing unsprung mass displacement will increase sprung mass acceleration.

Suspension designers have long appreciated the benefits of progressive springing and damping. Soft springs and little damping provide good isolation for small inputs, and stiffer springs and damping away from center prevent the suspension from crashing out its rattle space (15). If the valve drive calculated in Equation 5 is minimized as the suspension nears its maximum stroke, velocity goes continuously to zero as the suspension nears its maximum stroke, velocity goes continuously to zero as the actuator approaches maximum travel, essentially providing the same progressivity valued by suspension designers.

Similarly, the gain on the hub velocity in Equation 5 is a feedforward term. As it increases, the hub is undamped, and conversely as it is reduced, the hub is more damped. Lateral

![Figure 11. Root Locus of Hub Gains from 0 to 1](image_url)
velocity is measured on the vehicle, and hub damping can be a function of lateral velocity. In the presence of a lateral velocity, lateral force is developed at the contact patch of the tires. In this case, the hubs can be more damped, sacrificing sprung mass acceleration to improve unsprung mass deflection. Going straight down the highway, with minimal lateral acceleration, the hubs are undamped and sprung mass acceleration is minimized at the sacrifice of unsprung mass deflection. (This same trade-off was identified as possible with skyhook and groundhook inertial damping.)

In both of the cases, reducing open-loop actuator demand with stroke, and varying hub damping with lateral acceleration, nonlinearities in the control allow performance exceeding that possible with a linear optimal controller which formally balances the compromises.

**MODAL INERTIAL DAMPING**

So we have seen that inertial damping provides near optimal vertical vibration isolation, and allows a highly stable closed loop to be formed. Inertial damping does not attempt to influence vehicle handling. Conversely the Lotus modal control was shown to allow the ability to have the vehicle appear soft in heave, and more rigid for pitch and roll inertial inputs, and the ability to influence roll moment distribution. The Lotus modal control had relatively high single-wheel stiffnesses, provides relative heave damping, and has stability limits. Therefore these two general active suspension systems were in many ways opposite.

It is possible to combine inertial damping and Lotus modal control in way that preserves, and in some cases even improves upon the respective performance benefits of either. Given the hardware configuration of a corner of the Lotus suspension shown in Figure 9, the hydraulic servovalve is driven by the error between a demanded corner force and that measured in the load cell. Therefore a closed-loop force actuator is formed at each corner. In the actual implementation, this force loop was closed locally at each corner with analog compensation. This combination of analog compensation, no digital time delay, and high quality Moog servovalve, resulted in a high performance force actuator. Furthermore, the analog valve drive at each corner went through a multiplier chip so that digitally actuated end of travel stops were present.

With such a high quality force actuator, the Lotus hardware configuration could easily achieve independant skyhooks at each corner of the car. All that would be required is an accelerometer at each corner, whose output could be numerically integrated to yield inertial velocity of each corner. In order to preserve the benefits of the Lotus modal control, particularly in allowing distinctly parameterized responses to each rigid body mode of motion, inertial velocity at each corner could easily be transformed into heave, pitch and roll modal inertial velocities.

\[
C_{\text{heave}} = F_{1a} + F_{2a} + F_{3a} + F_{4a}
\]

**Equation 41**

\[
C_{\text{roll}} = \frac{1}{2} (F_{2a} + F_{3a}) - \frac{1}{2} (F_{1a} + F_{4a})
\]

**Equation 42**

\[
C_{\text{pitch}} = -a(F_{1a} + F_{2a}) + b(F_{3a} + F_{4a})
\]

**Equation 43**

where \( F_{ia} \) is the modal inertial damping active force at the \( i \)th corner.

Front and rear inertial roll dampings are defined by dividing Equation 43 into front and rear roll dampings.

\[
C_{\text{roll,front}} = \frac{1}{2} (F_{2a} - F_{3a})
\]

**Equation 44**

\[
C_{\text{roll,rear}} = \frac{1}{2} (F_{3a} - F_{4a})
\]

**Equation 45**

So that the total inertial roll damping of Equation 42 is the sum of the front and rear roll dampings.

\[
C_{\text{roll}} = C_{\text{roll,front}} + C_{\text{roll,rear}}
\]

**Equation 46**

Equation 46 does not shed insight on how to determine the front and rear inertial roll dampings \( C_{\text{roll,front}} \) and \( C_{\text{roll,rear}} \). For this we again use the roll moment distribution parameter \( \varepsilon \), where \( \varepsilon = 1 \) if all the roll moment is on the front, and \( \varepsilon = -1 \) if all at the rear. Using \( \varepsilon \) the values for front and rear roll stiffness can be written.

\[
C_{\text{roll,front}} = C_{\text{roll}} \left( \frac{1+\varepsilon}{2} \right)
\]

**Equation 47**

\[
C_{\text{roll,rear}} = C_{\text{roll}} \left( \frac{1-\varepsilon}{2} \right)
\]

**Equation 48**

Using Equation 41, Equation 42, Equation 47 and Equation 48 a relationship between modal accelerations and corner forces can be written.
Corner inertial velocities do not convey as much information as modal inertial velocities, and in fact contain redundancy, so in one sense Equation 54 does not have as much physical significance as Equation 36, the single wheel stiffness matrix of the Lotus modal control. However in another sense Equation 54 has great physical significance. The input vector of corner inertial velocities is measured - or more correctly integrated - from accelerometers at each corner. Therefore Equation 54 is the modal inertial damping control matrix from sensor measurements to actuator commands.

The first striking feature is that not only the similarity of the matrices of Equation 54 and Equation 36, but also the nomenclature used for simplification in Equation 37, Equation 38 and Equation 39, and Equation 51-Equation 53. The main diagonal of Equation 54 is seen to be similar to the main diagonal in Equation 36, the only difference being the replacement of the warp mode term by the roll moment distribution.

Despite the similarities, the modal inertial damping matrix of Equation 54 has significant differences from the Lotus modal control stiffness matrix of Equation 36. First, the Lotus modal control tried to shape second order dynamics of the various modes, but because they could not truly undamp the hubs, damping was relative and therefore desirably soft. In general the Lotus modal control worked best when the heave mode was soft and the pitch and roll modes were stiff. In the Lotus control modal stiffnesses added together to produce a single wheel stiffness, and that determined the responses to a single wheel event. Therefore despite the desire for a soft heave mode, for a single wheel event the stiff roll and pitch modes added to the single wheel stiffness, and the vehicle was perceived to be “harsh,” although the heave natural frequency was quite low. In the modal inertial damping matrix of Equation 54 the modal dampings individually add to form the single wheel inertial dampings on the diagonal. The difference is that modal damping is desirably high, so the addition of all the modal values for a single wheel value is not a problem. The high fidelity corner force loop deals with single wheel events as a loop disturbance to be rejected; much more effective than the Lotus modal control with the stiffness matrix of Equation 36.

Suspension determines handling as well as ride, and the inertial damping matrix of Equation 54 has significant handling advantages relative to the Lotus modal control. It was earlier shown in Equation 34 that the Lotus modal control roll moment distribution was determined by modal states and parameters. Thus, through judicious selection of the modal parameters, roll moment distribution could be affected, but only indirectly. In the modal inertial damping matrix of Equation 54, the roll moment distribution parameter ε is independently selectable.

A similar relationship between stiffness and actuator displacements is used, but in the modal inertial damping system Equation 54 has more authority.

Equation 49

\[
\begin{bmatrix}
\dot{\mathbf{z}} \\
\dot{\mathbf{\phi}} \\
\dot{\beta}
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
q_{\text{heave}} & -a & -a & -a \\
q_{\text{pitch}} & q_{\text{roll}} & q_{\text{pitch}} & q_{\text{roll}} \\
2q_{\text{roll}} & 2q_{\text{roll}} & 2q_{\text{roll}} & 2q_{\text{roll}} \\
0 & 0 & -t & 0
\end{bmatrix}
\begin{bmatrix}
F_{1a} \\
F_{2a} \\
F_{3a} \\
F_{4a}
\end{bmatrix}
\]

Equation 50

On the surface this seems to have been done by inverting a non-square matrix, however the four corner forces are still nonlinear functions of four variables, the three modal inertial velocities and the roll moment distribution.

Some nomenclature is introduced for simplification.

Equation 51

\[ C_h = \frac{abq_{\text{heave}}}{4(a+b)^2} \]

Equation 52

\[ C_r = \frac{q_{\text{roll}}}{4\varepsilon^2} \]

Equation 53

\[ C_p = \frac{q_{\text{pitch}}}{4(a+b)^2} \]

And using the definition of modal velocities a relationship between corner inertial velocities and corner forces can be constructed.

Equation 54

\[
\begin{bmatrix}
\frac{c_{1a} + (1+\varepsilon)c_d - c_c}{2} & \frac{c_{2a} - (1+\varepsilon)c_d + c_c}{2} & \frac{c_{3a} - (1+\varepsilon)c_d - c_c}{2} & \frac{c_{4a} - (1+\varepsilon)c_d + c_c}{2} \\
\frac{c_{5a} - (1+\varepsilon)c_d - c_c}{2} & \frac{c_{6a} + (1+\varepsilon)c_d + c_c}{2} & \frac{c_{7a} + (1+\varepsilon)c_d - c_c}{2} & \frac{c_{8a} - (1+\varepsilon)c_d + c_c}{2} \\
\frac{c_{9a} - (1+\varepsilon)c_d - c_c}{2} & \frac{c_{10a} + (1+\varepsilon)c_d + c_c}{2} & \frac{c_{11a} + (1+\varepsilon)c_d - c_c}{2} & \frac{c_{12a} - (1+\varepsilon)c_d + c_c}{2} \\
\frac{c_{13a} - (1+\varepsilon)c_d - c_c}{2} & \frac{c_{14a} + (1+\varepsilon)c_d + c_c}{2} & \frac{c_{15a} + (1+\varepsilon)c_d - c_c}{2} & \frac{c_{16a} - (1+\varepsilon)c_d + c_c}{2}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{2} \\
\frac{1}{2} \\
0 \\
\frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{z}} \\
\ddot{\mathbf{\phi}} \\
\ddot{\beta}
\end{bmatrix}
\]
between the absolute value of measured yaw rate and the yaw rate required for neutral steer based on the Ackermann relationship.

\[ \epsilon = K_{yaw} \left( |\gamma| - \frac{u}{a + b} \theta | \right) \]

**Equation 55**

Although the construction of the control of Equation 55 is relatively ad hoc, it has been shown to perform well relative to more rigorous nonlinear sliding mode controllers (17).

Road irregularities are not the only disturbances encountered by the vehicle suspension system. As the vehicle accelerates or decelerates, pitching moments are induced by inertia forces acting at the sprung mass of the vehicle through the lever arm of the height of the center of sprung mass. Similarly, as the vehicle turns, roll moments are induced. These moments can be calculated by knowing the mass properties of the vehicle, center of mass location, roll and pitch center locations, desired roll moment distribution, and measuring lateral and longitudinal accelerations. Although the center of mass changes with vehicle loading, by knowing static corner load values it can be easily located. Specifically,

\[ a_{mh} = a_{F_{14}} + a_{F_{24}} - b_{F_{12}} - b_{F_{24}}. \]

\[ \frac{(1 + \epsilon)}{2} a_{mh} = \frac{t}{2} \left( F_{14} - F_{24} \right), \]

\[ \frac{(1 - \epsilon)}{2} a_{mh} = \frac{t}{2} \left( F_{12} - F_{24} \right). \]

**Equation 56a-c**

where \( a_x \) is longitudinal acceleration, \( a_y \) is lateral acceleration, \( h \) is the roll and pitch center heights, and \( F_i \) is the feedforward force associated with the \( i \)th corner.

Equation 56a-c, in addition to the trivial equation that the sum of all feedforward forces must be zero can be written in matrix form as

\[
\begin{bmatrix}
\frac{mha_x}{nb(1+\epsilon)} a_x \\
\frac{mhb_y}{nb(1-\epsilon)} a_y
\end{bmatrix}
= \begin{bmatrix}
a & a & -b \\
1 & -1 & 0 \\
0 & 0 & -1 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
F_{14} \\
F_{24} \\
F_{12}
\end{bmatrix}.
\]

**Equation 57**

Inverting this system of linear equations the feedforward corner forces can be obtained in terms of the lateral and longitudinal accelerations and the roll moment distribution, that is,

![Figure 12. Composite Driver Discomfort vs. Controller Effort](image-url)
Note that Equation 58 can be used with the modal inertial damping and independent inertial damping controllers. The feedforward force terms expressed in Equation 58 are added to the closed-loop force demands, $F_{1a}$, $F_{2a}$, $F_{3a}$ and $F_{4a}$ in Equation 54 so that

$$F_1 = F_{1a} + F_{1f} ,$$

$$F_2 = F_{2a} + F_{2f} ,$$

$$F_3 = F_{3a} + F_{3f} ,$$

$$F_4 = F_{4a} + F_{4f} .$$

**Equation 59a-d**

where $F_i$ is the total force on the $i$th corner, and is the summation of the modal inertial damping active force and the feedforward force of the $i$th corner.

It is interesting to note that using the feedforward control of Equation 59a-d in conjunction with the independent inertial damping would yield near optimal ride isolation and still allow some dynamic roll moment distribution for handling. For optimal handling while maintaining very good ride isolation modal damping is still preferred (18). Roll moment distribution derived from modal inertial damping has authority for transient maneuvers, while roll moment distribution derived from acceleration feedforward has authority for steady state conditions.

**COMPARISON WITH OPTIMAL CONTROLLER**

An optimal controller can be constructed to compare with modal inertial damping. The active suspension ride problem sets up very well for a standard LQR/LQG optimal control formulation as minimizing rms accelerations of a driver located on the sprung mass is an accepted performance criteria, and vertical road inputs are stochastically well defined (19). A model was constructed including 4 unsprung masses and 3 rigid body modes, yielding 14 mechanical states. Actuator dynamics were shown to be important, and second order valve dynamics and oil compliance at each corner adds 12 hydraulic states, for a total of 26 states (20).

Driver discomfort can be calculated knowing the driver's location relative to the vehicle center of gravity and human sensitivity to vibration along the various axes. Inertial modal damping gains were selected to resemble the driver discomfort coefficients of the various modes.

The cost of control in terms of rms actuation can be varied from a relatively expensive control to cheap control, serving to define an envelope of possible controllers. This is a particularly revealing approach for broad bandwidth active suspension as the control variable is valve drive, and therefore
proportional to power consumption. This envelope can be generated for the optimal full state feedback LQR controller, and is considered the benchmark against which all other controllers can be measured. This envelope of achievable isolation performance for various actuator efforts can also be generated for independent skyhook control and inertial modal damping as shown in Figure 12.

Because actuator dynamics were included, the independent skyhook controller and the modal inertial damping controller both have a stability limit as the control effort grows. In Figure 12 the inertial damping curves are terminated to preserve a 6dB stability margin. In practice it is not anticipated that the LQR controller could continue to achieve marginally better performance with more control effort, as unmodeled dynamics are often excited when higher control effort is expended. It is apparent that the independent skyhook performs a bit better in reducing transmissibility of road noise. However, the inertial modal control algorithm performs well, and still allows independent specification of roll moment distribution.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>NASA Ride Index</th>
<th>Active Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – Modal Inertial Damping</td>
<td>2.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Best in Class passive</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>B – passive luxury ride</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>C – Active</td>
<td>2.9</td>
<td>0.3</td>
</tr>
<tr>
<td>Luxury import</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>B - Active</td>
<td>3.1</td>
<td>0.9/-0.2</td>
</tr>
<tr>
<td>Domestic luxury</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>C - Passive</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>A – Lotus Modal Damping</td>
<td>3.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Domestic luxury</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>B – passive sport</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>Domestic midsize sport</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>A - Passive</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

Figure 14: Various Vehicles' Objective Ride

Figure 15: Steering Input for Various Roll Moment Distributions

Until reaching its stability limit, the modal inertial damping approach performs better than the independent skyhooks, midway to the benchmark LQR controller. Modal inertial damping and the LQR algorithms can influence pitch and roll motions in ways that the independent skyhooks cannot.

As expected, the independent skyhook dampers do a better job of isolating the heave mode as shown in Figure 13. This makes sense, as heave is the only information used by the independent damping algorithm, so the entire authority and bandwidth of the force actuators in that system is used to isolate heave. In the modal inertial damping system pitch and roll are also controlled. As it turns out, based on the driver discomfort index selected, both the LQR and the inertial modal control placed heavy emphasis on controlling pitch.

VEHICLE RESULTS

The modal inertial damping algorithm was implemented on the same passenger car that previously had the Lotus modal control. This, along with several other active and passive vehicles, was evaluated using the NASA ride index that frequency weights vibrations in several modes, very similar to the driver discomfort function used in the previous section. A difference of 0.2 in the ride quality is generally perceptible.

The passive version of Vehicle A has a NASA ride quality of 4.5; not a particularly good riding car among the data displayed, and worse than a couple comparable competing midsize sport models. The Lotus modal damping algorithm improved Vehicle A's ride quality to 3.3, which is comparable to luxury models. Modal inertial damping further improved the ride quality of Vehicle A to 2.6, notably exceeding the ride quality of the best in class passively suspended vehicle.

Vehicle B featured a competing active system that demonstrated better ride that the passive sport version, but not as good ride as the passive luxury version. Another Vehicle C showed a slight improvement in active ride quality. No information is available on the control strategies used in these vehicles.
Taken together, this data tends to support the perception that prior to modal inertial damping active suspension can improve an average ride, but not exceed a good ride.

The effect of varying the roll moment distribution can be seen on an actual vehicle, where the vehicle's trajectory was constrained to a 90 degree turn at 30 mph. The passive vehicle required a large handwheel input to negotiate the defined trajectory as shown in Figure 15. Varying degrees of static roll moment distribution from 1.0 (all on the front) to -1.0 (all on the rear) show that as the roll moment moves to the rear, the car becomes more oversteering and less handwheel input is required. Also shown in Figure 16 is the roll moment distributed according to the absolute yaw rate error of Equation 55.

The time history of the dynamic roll moment distribution parameter shown in Figure 16 reveals interesting closed-loop behavior. At the beginning of the turn, the roll moment is biased to the rear, and one point saturating at -1.0. Throughout the middle portion of the turn the roll moment is relatively constant at approximately -0.5. Finally, upon exit of the turn the roll moment is biased to the front, allowing the rear wheels to bite and stabilize the vehicle in its straight ahead position.

The driver's impression of this vehicle is quite dramatic. It is virtually impossible for this vehicle to spin out on a curve. If a driver enters a curve too fast, the car does not lose grip at the front (understeer) or the rear (oversteer), rather the car drifts laterally away from the trajectory that requires more lateral force than the tires can generate.

Taken together, the improvements in ride and handling of the inertial modal control produced the best riding vehicle measured to that date, as well as dramatic direct closed-loop control of yaw rate to assure neutral steer (or any desired degree of understeer). Such a simultaneous improvement in both ride and handling, properties historically considered in conflict, was unprecedented.

INERTIAL VELOCITY CALCULATION

The dramatic improvement in ride data shown in Figure 14 was generated by a vehicle with modal inertial damping. Modal damping requires inertially referenced velocities at each corner of the car, as seen in Equation 54. The most economical way to construct an inertial velocity signal is to integrate accelerometers, which are inertially referenced.

In theory integration of accelerometer signals is easily done, however in practice inertially referenced velocity proves to be as elusive as it is useful. Unfortunately simple integration will not suffice as such accelerometers are inevitably subject to bias, drift, and output sensitivity due to sensor orientation. These inevitable low frequency errors are multiplied by desirable high inertial damping gains resulting in very large low frequency force errors.

The classic solution to accelerometer integration difficulties has been pseudo-integration, or the use of filters resembling theoretical integrators in the frequency range of interest, while having desirable bias and drift rejection properties at lower frequencies (21,22). It is possible to use a first order low-pass filter to mimic the behavior of an integrator -- -20 dB/decade roll off and 90 degree phase lag -- in the frequency range of interest, in this case the resonant ride frequency. Such a first order filter has a finite steady-state gain so that a biased input signal will result in a non-zero filter output. This non-zero filter output is multiplied by the large skyhook damping term to produce a force that will move the suspended mass until it is balanced by the force created by load spring deflection. For good isolation the load spring constant is typically small. The combination of high inertial damping gain and low load spring constant makes such systems extremely vulnerable to low frequency errors in the inertial velocity signal. To remedy this situation second order band-pass filters have typically been used. The transfer function of this filter can be written as

\[ G_f(s) = \frac{s}{s^2 + 2\zeta \omega s + \omega^2} \]

\[ \text{Equation 60} \]

where \( \omega \) is the natural frequency of the filter and \( \zeta \) is the damping ratio. A pseudo-integrator is formed if \( \omega \) is sufficiently low relative to the mechanical resonance and \( \zeta \) is around 0.707. It is evident from Equation 60 that the filter behaves as an integrator at frequencies sufficiently greater than the filter natural frequency while it attenuates low frequency drifts and eliminates steady state bias. The transfer function relating a single degree of freedom sprung mass acceleration to an actuator force can be readily expressed where \( M \) is the sprung mass and \( K \) is the suspension spring constant.
When the filter of Equation 60 is applied to the sprung mass acceleration output of Equation 61, the open-loop frequency response of force input to filter output is shown in Equation 62 and illustrated in Figure 17.

\[ \frac{\ddot{x}_{\text{fil}}}{F} = \frac{s^2}{(Ms^3 + K)} \]

Equation 61

When the filter of Equation 60 is applied to the sprung mass acceleration output of Equation 61, the open-loop frequency response of force input to filter output \( \ddot{x}_{\text{fil}} \) is shown in Equation 62 and illustrated in Figure 17.

\[ \frac{\ddot{x}_{\text{fil}}}{F} = \frac{s^3}{(Ms^3 + K)(s^2 + 2\zeta\omega s + \omega^2)} \]

Equation 62

Figure 17 significantly differs from Figure 5 in that at low frequencies the open-loop phase passes through 180 degrees of lead. At frequencies greater than the mechanical resonance(s), the filtered open-loop of Figure 17 and the ideal open-loop of Figure 5 exhibit similar desirable stability properties expected of positive real systems. The critical phase crossover of the filtered open-loop of Figure 17 is determined not by the customary 180 degree phase lag point, but rather the 180 degree phase lead frequency, in this case about 3 rad/sec. This low frequency cross-over of the filtered system is exclusively a function of the band-pass pseudo-integrating filter, as long as the filter natural frequency, \( \omega \) in Equation 60, is small relative to the sprung mass resonant frequency of Equation 61 - a design criteria of the band-pass pseudo-integrating filter. Even though these approximate integrators possess more robust low frequency properties than pure integrators, performance is constrained by a low frequency instability property of practical skyhook damping systems not fully appreciated in the control theory community at the time. For example an excellent classic text (23) defines gain margin in terms of a 180 degree phase lag with no mention of lead:

\[ G(j\omega) = \frac{s}{s^2 + 2\zeta\omega s + \omega^2} \]

Equation 63

The gain margin is the reciprocal of the magnitude \( |G(j\omega)| \) at the frequency where the phase angle is -180 degrees.

The analysis to this point has neglected actuator dynamics. As lags in actuator dynamic response become significant, high frequency phase lag of either the ideal inertial damping system or the filtered system will produce a traditional 180 degree phase lag cross-over frequency as shown in Figure 6. Thus the practical system does not enjoy the positive real stability properties of the earlier idealized system.

For reasonable actuator dynamics, the low frequency cross-over determines the closed-loop gain level, and therefore the amount of inertial damping that can be applied, and consequently overall system performance. Thus when sufficiently high performing force actuators are used, loop gain is limited by the filtering process required to remove bias and drifts from the accelerometer signals upon integration to form velocity. It is inefficient to allow low frequency linear filter dynamics to limit control system performance as hardware bandwidth is under-utilized. Experience has shown that even the standard 6 dB gain margin is unacceptable in active suspension applications as low frequency road undulations are amplified, resulting in a sprung mass oscillation perceptibly "disconnected" from the road.

As seen in Figure 5, if a pure inertially referenced velocity signal were available, the resulting system would be extremely stable, limited only by dynamic lags from force actuator dynamics. As the intent of the system is to provide very high levels of inertial damping, the filter should not transfer a biased input signal, and should have good low frequency drift rejection properties.

\[ \ddot{x}_{\text{fil}} = G_{f_1}\dot{x}_{\text{out}} + G_{f_2}x_{\text{out}} \]

\[ G_{f_1} = \frac{s}{s^2 + 2\zeta\omega s + \omega^2} \]

\[ G_{f_2} = \frac{2\zeta\omega s}{s^2 + 2\zeta\omega s + \omega^2} \]

Equation 63

Equation 63 describes transfer functions for complementary filters on acceleration and position which, when added together, result in inertial velocity. The acceleration filter \( G_{f_1} \) is identical to Equation 60 which by design has favorable drift and bias rejection properties. By virtue of its zero at 0 rad/sec, the displacement filter \( G_{f_2} \) has similar robust drift and bias rejection qualities. Furthermore, the displacement filter of Equation 63 is seen to be well
behaved at high frequencies, unlike a simple differentiator of displacement which is subject to high frequency noise amplification (24). The disadvantage of Equation 63 is that for strict implementation it requires inertially measured position $x_{out}$, however it is perhaps more economical to use relative position.

The inertial velocity calculation of Equation 63 was implemented on a single degree of freedom system designed to inertially damp the pitching motion of a semi-tractor cab (25). The goal of the system was to actively damp the pitch mode, and a premium air suspended seat would isolate heave, thereby providing high quality pitch-plane ride performance. As shown in Figure 18, an inertial damping system using Equation 60 to calculate inertial velocity significantly improves the pitch acceleration felt by the driver. When the complementary filter of Equation 63 is used, the damping is reduced even more. In fact, using the complementary system the resonant pitch acceleration is reduced to roughly the level of the background noise.

Unfortunately, the complementary filter was invented after work on the broad bandwidth active system had stopped, so its potential influence on the full vehicle ride quality of Figure 19 is somewhat speculative (26). It is known from vehicle measurements that the inertial pitch damping system resulted in an average improvement of NASA ride quality of approximately 0.99. From Figure 18 it is evident in the only mode controlled, that the complementary system provides at least 50% improvement beyond the band pass system that was similar to the multi-degree of freedom algorithm used in Figure 14. Therefore it is reasonable to assume the complimentary filter provided 0.5 improvement in the NASA ride quality index. If that performance improvement could be duplicated on the modal inertial damping vehicle reported in Figure 14, it would be transformed from marginally best-in-class to significantly best-in-class.

This projection can be considered conservative for two reasons. First, the half-point improvement attributed to the complementary filter effect was due to the overall ride quality improvement measured compared to the bandpass system. But overall ride quality is comprised of accelerations in all rigid body directions, and the control system investigated only improved pitch. Therefore, it is reasonable to expect that if heave were similarly improved, as it would be in the modal inertial damping vehicle, the ride quality improvement would be even higher.

Second, the active cab pitch control system was realized with lower bandwidth actuators than those used on the modal inertial damping vehicle. The effect of the complementary filter is to eliminate the low frequency instability due to the pseudo-integrators, leaving the closed-loop performance determined by the stability at high frequencies, and therefore determined by actuator dynamics. The higher bandwidth actuators of the modal inertial damping Vehicle A of Figure 14 should allow more improvement when the complementary filter of Equation 63 is used.

**CONCLUSION**

Active suspension development has been an interesting combination of many concepts: ride and handling, vehicle...
dynamics and controls, theory and practice, linear and nonlinear, optimal and clever. Because of the nature of the dynamics, active suspension has been the subject of much theoretical attention. Because of the significant expense of hardware development, and the depth of required expertise in a number of areas, hardware implementations have been far less common. And yet, as with many truly interesting systems, it is the practical realities that often offer the most profound theoretical challenges.

The early work by Lotus was truly pioneering. Just as Maurice Olley and Bill Milliken transferred aerospace engineering techniques to automotive, so did David Williams and Peter Wright. Together they formed an algorithm combining general flight control techniques and specific automotive nonlinearities. Sensor measurements, most significantly force, were converted to velocity demands at each corner of the vehicle, and those velocities were achieved by driving high precision servovalves open-loop. The result was a solution providing ride benefits to most vehicles, and handling benefits to all, including Formula One.

In parallel to these efforts, Dean Karnopp and Don Margolis from UC Davis came up with the notion of skyhook damping, taking advantage of the particular dynamic composition of the problem that offered extreme stability when force actuators were driven with inertial velocity signals. Skyhook damping was a concept most appreciated by control theorists focused on ride isolation.

At the joint venture between Moog and Lotus, that was eventually purchased by TRW, inertial damping was combined with the modal control of Lotus. The equations forming the corner velocity demands from measured forces of the Lotus algorithm were "turned inside out." In the spirit of skyhook damping, measured velocities were used to form forces at each corner. In between however, the measured velocities were transformed to modal velocities, modal stiffness and dampings applied to yield modal forces, and then the modal forces transformed back to corner forces. Thus the highly stable inertial damping principle of Karnopp was combined with independently tunable rigid body modes of Lotus.

Force actuators at each corner were closed-loop, and thus able to absorb single wheel inputs more effectively than the open-loop velocities at each corner provided by the Lotus algorithm. Provision was made in the electronics at each corner to preserve key nonlinearities that were so useful in the Lotus system. This control strategy provided best in class ride for the first time in any active car.

The implementation of modal inertial damping allowed for excellent ride quality, but also provided unprecedented handling. The Lotus algorithm influenced roll moment distribution indirectly through its warp mode dynamics. Modal inertial damping allowed direct specification of roll moment distribution, and used it for closed-loop control of yaw rate to assure predictable handling.

The factor limiting the ride performance of modal inertial damping was not actuator dynamics, as would be expected in a high performance control system. Rather the instability was at low frequencies due to the pseudo-integrators used to get inertial velocity from acceleration. This problem was diagnosed and a solution found after work on the broad bandwidth active system stopped. It was conservatively estimated that if these new complementary integrators were used, modal inertial damping would be significantly better than the previous best in class.

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REFERENCES


25. Williams, Dan, et. al., “Low Bandwidth Active Cab Suspension”, SAE paper 973206.