Control of sound
with periodic structures

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ASME NCAD Tutorial
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Why should NCAD mechanical engineers be interested?

potential for radically new devices/technologies
- vibration insulation
- ultrasonic imaging beyond the diffraction limit
- filtering, guiding, ...
- stealth for underwater structures ....
OUTLINE

Extraordinary acoustic transmission

Extraordinary acoustic absorption

Phononic crystals – some theory

Negative index materials

Transformation acoustics - cloaking
1D periodic media

extraordinary acoustic transmission
1D periodic media

extraordinary acoustic transmission
\[ R = \frac{i}{2} \left( \frac{Z_\theta}{Z_\theta'} - \frac{Z_\theta'}{Z_\theta} \right) T \sin \frac{\omega a}{c_\theta} \]

\[ Z_\theta = \frac{\rho c}{\cos \theta} = \frac{z}{\cos \theta} \]

\[ c_\theta = \left( \frac{\rho_s}{K_s} - \frac{\sin^2 \theta}{c^2} \right)^{-\frac{1}{2}}, \quad Z_\theta' = \rho_s c_\theta = z_s \sqrt{1 - \frac{c_s^2}{c^2} \sin^2 \theta} \]

**Full transmission at all frequencies if**

\[ Z_\theta = Z_\theta' \]

\[ |T(\theta)| = 1 \quad \text{↔} \quad \tan^2 \theta = \frac{1 - \frac{z^2}{z_s^2}}{\frac{c_s^2}{c^2} - 1} \]
**Transparent slab - at one angle**

\[ R = \frac{i}{2} \left( \frac{Z_\theta}{Z'_\theta} - \frac{Z'_\theta}{Z_\theta} \right) T \sin \frac{\omega a}{c_\theta} \]

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Full transmission at all frequencies if \[ Z_\theta = Z'_\theta \]

\[ |T(\theta)| = 1 \quad \leftrightarrow \quad \tan^2 \theta = \frac{1 - \frac{\rho_s^2}{c_s^2}}{\frac{c_s^2}{c^2} - 1} \]

“Brewster angle”
\[ \frac{1}{K_s} = \frac{f}{K_0} + \frac{1-f}{K} \]

\[ \frac{1}{\rho_1} = \frac{f}{\rho_0} + \frac{1-f}{\rho} \]

\[ \rho_2 = f \rho_0 + (1-f) \rho \]

“homogenize”
at all frequencies if

\[
\left| T(\theta) \right| = 1
\]

\[
\tan^2 \theta = 1 - \frac{K_s \rho_1}{K_s \frac{z^2}{c^2 \rho_2} - 1}
\]

\[
\frac{1}{K_s} = \frac{f}{K_0} + \frac{1-f}{K}
\]

\[
\frac{1}{\rho_1} = \frac{f}{\rho_0} + \frac{1-f}{\rho}
\]

\[
\rho_2 = f \rho_0 + (1-f) \rho
\]

(D’Aguanno et al. 2012; Maurel et al. PRB 2013)
Transparent slab - at one angle

\[ |T(\theta)| = 1 \quad \text{at all frequencies} \quad \text{if} \quad \tan^2 \theta = \frac{1 - \frac{K_s \rho_1}{c^2 \rho_2}}{\frac{K_s}{c^2 \rho_2} - 1} \]

\[ \frac{1}{K_s} = \frac{f}{K_0} + \frac{1-f}{K} \]

\[ \frac{1}{\rho_1} = \frac{f}{\rho_0} + \frac{1-f}{\rho} \]

\[ \rho_2 = f \rho_0 + (1 - f) \rho \]

(Maurel et al. PRB 2013)

anisotropic density
Full transmission *at all frequencies* if

$$\tan^2 \theta = \frac{1}{\frac{K_s \rho'_1}{K_s c^2 \rho'_2} - 1}$$

$$\frac{1}{K_s} = \frac{f}{K_0} + \frac{1-f}{K}$$

$$\frac{1}{\rho'_1} = \frac{\cos^2 \phi}{\rho_1} + \frac{\sin^2 \phi}{\rho_2}$$

$$\rho'_2 = \rho_2 \cos^2 \phi + \rho_1 \sin^2 \phi$$

(Norris 2014)
Full transmission \textit{at all frequencies} if \[\tan^2 \theta = \frac{1 - \frac{K_s \rho_1'}{\rho_2'}}{c^2 \rho_2'} - 1\]

\textbf{Rigid immovable limit:} \[\cos \theta = (1 - f) \cos \phi\]

\textbf{Intromission angle}
periodic media

extraordinary acoustic absorption
low frequency absorption

cubic array of lead coated spheres + layer of silicone rubber

very low frequency band gap

Z. Liu et al Science 2000
Tuned mass damper

www.acs.psu.edu/drussell/demos.html
point mass on a bar
point mass on a bar

$u_{\text{inc}} = 1$  \hspace{1cm} R \hspace{1cm} T
point mass on a bar

\[ u_{\text{inc}} = 1 \]

\[ \sigma(0+, t) - \sigma(0-, t) = m\ddot{u}(0, t) \]
point mass on a bar

\[ u_{inc} = 1 \quad R = 1 - T \quad T = \frac{1}{1 - \frac{i\omega}{2\zeta} m} \]

\[ \sigma(0^+, t) - \sigma(0^-, t) = m\ddot{u}(0, t) \]
resonant mass on a bar

\[ \Omega = \frac{\omega}{\omega_r} \]

\[ |T|^2 = \frac{1}{1 + \left( \pi \frac{m}{M} \frac{\Omega}{1 - \Omega^2} \right)^2} \]

\[ M = \text{mass of one wavelength of bar at resonance} \quad (\Omega = 1) \]

(after B.R. Mace 2014)
resonant mass on a bar

\[ M = \text{mass of one wavelength of bar at resonance} \quad (\Omega = 1) \]

\[ |T|^2 = \frac{1}{1 + \left(\pi \frac{m}{M} \frac{\Omega}{1 - \Omega^2}\right)^2} \]

\[ \Omega = \frac{\omega}{\omega_r} \]
low frequency band gap: heavy & soft

Lead coated spheres + layer of silicone rubber in cubic array

very low frequency band gap and negative elastic constant due to dipole resonance

sub-wavelength band gap \( \lambda \gg a \)

negative effective mass

mass-in mass resonant oscillator system

equivalent system

compare equilibrium eqns:

\[
\frac{m_{\text{eff}}}{m_{\text{st}}} = 1 + \left(\frac{\theta}{1+\theta}\right) \frac{(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2}.
\]

\[
\theta = \frac{m_2}{m_1} \quad \omega_0^2 = \frac{k_2}{m_2} \\
\]

\[
m_{\text{st}} = m_1 + m_2
\]

Negative effective mass for

\[
1 < (\omega/\omega_0)^2 < 1 + \theta.
\]

Control bandgap frequencies

H. H. Huang et al 2009
Stacked panels with different masses

15 kg/m²

30 cm

Broadband transmission loss

Transmission Loss of membrane-type metamaterials

Mass law \( \text{TL} \propto \log_{10} \text{thickness} \times \text{frequency} \)

M-T M \( \text{TL} \propto \text{thickness} \)

super absorber

unit cell: elastic membrane on a rigid grid with asymmetric iron platelets

material absorbs 70% plus of acoustic energy

transmission data for 200 μm membrane

wave metamaterial

flapping platelets absorb acoustic energy

symmetrical platelet design

phononic crystals

2D and 3D
phononic crystals

= periodic mechanical systems

phononic band gaps provide tools for controlling waves:

• filtering
• absorbing
• steering
• trapping, etc.

Review:
phononic crystals: filtering and controlling waves

perfect mirror (in band gap)
- frequency filter

and waveguides ("tubes")

can trap waves in cavities

Khelif et al (APL 2004)

wave guide
- beam splitting, multiplexing
surface acoustic waves

Mohammadi et al. (APL 2008)

hexagonal

cubic
phononic crystals in engineering applications

**acoustic insulation** e.g. sound barriers
inexpensive materials

**low frequency** vibration isolation,
using internal resonators

**waveguides, SAW filters**
great potential, possible to fabricate,

**negative index materials, lens** work well in theory
could provide super focusing, e.g. biomedical imaging

(Sanchez-Dehesa et al. ’10)

(Lin et al. JAP ’09)

(Croenne et al. ’11)
2D sonic crystals with tailored properties

lensing effect

scattering from circular array of cylinders

equivalent material

gradient index lens

plane wave focusing

Sánchez-Dehesa, Torrent, L-W Cai (NJP 2009)

Martin et al. (APL 2010)
mechanism of sonic crystal focusing

dense solid cylinders in lighter fluid $\rightarrow$ effective medium with $v_{eff} < v_{acoustic}$
phononic crystals can lead to **negative refraction**

sonic crystal

Positive refraction

\[ \lambda \gg a \]

phononic crystals

Negative refraction

\[ \lambda \approx a \]

potential for resolution beyond the diffraction limit

“perfect lens”
phononic crystals can lead to **negative refraction**

sonic crystal

Positive refraction

\[ \lambda \gg a \]

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Negative refraction

\[ \lambda \approx a \]

potential for resolution beyond the diffraction limit

“perfect lens”
phononic crystals

theory
scattering of sound

\[ p \approx Ae^{i(k \cdot x - \omega t)} \]
periodic system

\[ \lambda = \frac{2\pi}{k} \]

for most \( \lambda \), sound propagates through crystal \textit{without} scattering

(scattering cancels \textit{coherently})

...but for some \( \lambda (~ 2a) \), no sound propagates: \textit{a phononic band gap}
waves in periodic systems

Bloch-Floquet theorem:

\[ u(x, t) = e^{i(k \cdot x - \omega t)} U_k(x) \]

- Plane wave
- Periodic envelope

\( k = \text{constant}, \ i.e. \ no \ scattering \ of \ Bloch \ wave \)

\( U_k \leftrightarrow \) modes of the finite unit cell, i.e. discrete frequencies \( \omega_n(k) \)
Any 1D periodic system has a gap

band gap: range of frequency in which no wave propagates

First, treat as artificially periodic

Bands "folded" by $2\pi/a$ equivalence

$\rho_1$

$\rho(x) = \rho(x+a)$
Any 1D periodic system has a gap

consider uniform medium as “artificially” periodic

forward, backward propagating waves or standing waves

\[ u = u_0 \cos(kx - \omega t) + u_0 \cos(kx + \omega t) = 2u_0 \cos kx \cos \omega t \]
Any 1D periodic system has a gap

add small *periodic* density $\rho_2 = \rho_1 + \Delta \rho$

splitting: wave mainly in slower zone ($\rho_2$) has lower frequency

$$U = \frac{\omega}{k}$$

same $k = \frac{\pi}{a}$

$\omega$ vs $x = 0$ to $\pi/a$

band gap

$\sin\left(\frac{\pi}{a} x\right)$

$\cos\left(\frac{\pi}{a} x\right)$

$\rho(x) = \rho(x+a)$
simplest 1D periodic system

Newton’s chain

\[-m\omega^2 u_n = \kappa (u_{n+1} - u_n) + \kappa (u_{n-1} - u_n)\]

\[u_n = u_0 e^{i(nka - \omega t)}\]

\[\cos ka = 1 - \frac{\omega^2}{2\omega_0^2}\]

\[\omega_0^2 = \frac{\kappa}{m}\]
simplest 1D periodic system

Newton’s chain

\[-m \omega^2 u_n = \kappa(u_{n+1} - u_n) + \kappa(u_{n-1} - u_n)\]

\[u_n = u_0 e^{i(nka - \omega t)}\]

\[\frac{\omega}{\omega_0} = 2 \sin \frac{ka}{2}\]
next simplest 1D periodic system

Born chain

\[ m_1 \kappa m_2 \]

\[ \alpha \]

\[ u_n \]

\[ \cos^2 k a = \left( 1 - \frac{\omega^2}{2\omega_1^2} \right) \left( 1 - \frac{\omega^2}{2\omega_2^2} \right) \]

\[ u_{2n} = u_0 e^{i(k2a - \omega t)} \]

\[ u_{2n+1} = u_1 e^{i(k2a - \omega t)} \]

\[ \omega_1^2 = \frac{\kappa}{m_1} \]

\[ \omega_2^2 = \frac{\kappa}{m_2} \]

"optical"

\[ \frac{\omega}{\omega_0} \]

\[ \omega_0 = \frac{1}{2} (\omega_1^2 + \omega_2^2) \]

"acoustic branch"

"band gap"
Bloch waves – 1D periodic system

example: use propagator matrix

\[ \frac{\rho \dot{v}}{\rho} = -\nabla p \]
\[ \dot{p} = -K \text{div } v \]

\[ p = p(x)e^{-i\omega t} \]
\[ v = v(x)e^{-i\omega t} \]

\[ \begin{pmatrix} v' \\ p' \end{pmatrix} = i\omega \begin{pmatrix} 0 & K^{-1} \\ \rho & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix} = i\frac{\omega}{c} \begin{pmatrix} 0 & z^{-1} \\ z & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix} \]

ordinary differential equation

\[ -i\omega \rho v = -p' \]
\[ -i\omega p = -K v' \]

where

\[ K = zc \]
\[ \rho = z/c \]

impedance \( z = \sqrt{K\rho} \)
Bloch waves – 1D periodic system

Example: use **propagator matrix**

\[
p = p(x) e^{-i\omega t}
\]
\[
v = v(x) e^{-i\omega t}
\]

\[
\begin{pmatrix}
  v' \\
  p'
\end{pmatrix} = i\omega 
\begin{pmatrix}
  0 & K^{-1} \\
  \rho & 0
\end{pmatrix}
\begin{pmatrix}
  v \\
  p
\end{pmatrix}
\]

\[
\rho \dot{v} = -\nabla p
\]
\[
\dot{p} = -K \text{ div } v
\]

\[
k = \frac{\omega}{a}
\]

\[
\eta(x) = \begin{pmatrix}
  v \\
  p
\end{pmatrix}
\]

\[
A = \begin{pmatrix}
  0 & z^{-1} \\
  z & 0
\end{pmatrix}
\]

\[
\eta' = ikA\eta
\]

Impedance \( z = \sqrt{K\rho} \)

Example: uniform medium

\[
\eta(x) = e^{ikx}A\eta(0)
\]
Bloch waves – 1D periodic system

\[ k = \frac{\omega}{a} \]
\[ \eta(x) = \begin{pmatrix} u \\ p \end{pmatrix} \]
\[ A = \begin{pmatrix} 0 & z^{-1} \\ z & 0 \end{pmatrix} \]

ordinary differential equation
\[ \eta'(x) = i k A \eta \]

uniform medium:
\[ \eta(x) = e^{i k x A} \eta(0) \]
\[ e^{i k x A} = I \cos kx + i A \sin kx \]

propagator

Example: 2 phase medium

Bloch condition
\[ \eta(a) = e^{i k a} \eta(0) \]
\[ e^{\pm i k a} \]

dispersion relation
\[ \cos k a = \cos \frac{\omega}{c_1} a_1 \cos \frac{\omega}{c_2} a_2 - \sin \frac{\omega}{c_1} a_1 \sin \frac{\omega}{c_2} a_2 \frac{1}{2} \left( \frac{z_2}{z_1} + \left( \frac{z_1}{z_2} \right) \right) \]
Bloch waves – 1D periodic system

<table>
<thead>
<tr>
<th>layer</th>
<th>thickness</th>
<th>density</th>
<th>stiffness</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>1</td>
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<tr>
<td>2</td>
<td>0.313</td>
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<td>7</td>
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<td>0.317</td>
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first 4 Bloch wave branches

$$\omega$$

$$\pi$$

$$\frac{k}{\pi}$$

fundamental

$${}^{3}$$rd

$${}^{2}$$nd
Bloch waves – 1D periodic system

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first 4 Bloch wave branches

\[ \frac{d\omega}{dk} < 0 \]

negative group velocity

???
phase and group velocities

\[ e^{i(kx - \omega t)} \]

phase \[ v \equiv \frac{\omega}{k} \]

group \[ v_g \equiv \frac{d\omega}{dk} \]
negative group velocity is not unusual

periodic systems

plates

and rods

Lamb 1904
Lamb’s model of negative group velocity


first “negative stiffness/metamaterials” paper ??

\[ \frac{\partial^2 u}{\partial t^2} + p^2 u + c^2 \frac{\partial^2 u}{\partial x^2} = 0 \quad c^2 = \frac{p}{\rho} \]

\[ u = A \cos(kx - \omega t) \quad \Rightarrow \quad \omega^2 = p^2 - c^2 k^2 \]

\[ \nu \nu g = -c^2 \]
dispersion

\[ \nu < \nu_g \neq \text{constant} \]
negative group velocity, positive phase velocity
phase and group velocities
phononic crystals

negative index materials
phononic crystals can lead to **negative refraction**

- sonic crystal
- phononic crystals

---

Positive refraction

$\lambda >> a$

Negative refraction

$\lambda \approx a$

potential for resolution beyond the diffraction limit

“perfect lens”
solid matrix negative index materials

**objective**: negative index material for sound in water
e.g. improved imaging for biomedical applications

**Metal matrix**

**Morvan et al.** APL 2010

**Epoxy matrix**

**Croenne et al.** PRB 2011
negative index for sound in water

negative group velocity effect in phononic crystal

phase matching to negative group velocity

ultrasonic negative index lens at 0.55 MHz

Sukhovich, Jing, Page (PRB 2008)
solid matrix device

steel rods embedded in epoxy – triangular lattice


efficient coupling to sound in water remains a challenge
“Metal Water structure”

- Negative group velocity at sonic speed
- Density of water (by design)
- Almost circular equi-frequency contour

Bulk modulus = 2.25 GPa
Density = 1000 kg/m³
Shear modulus = 0.065 GPa (i.e. small)

Flat lens

Homogenize
MW FLAT LENS - simulation

Negative refraction of longitudinal waves

Distance source/lens : 8.9 mm

Numerical results : index matching 😊
                     impedance matching 😞
                     position of the image = 😊

Lateral resolution : width at half maximum = 0,59 λ

Prototype: 15 glued plates, cut by water jet

top view before adding solid plate

solid plate on the bottom

MW FLAT LENS - measurement

transformation acoustics

acoustic cloaking
transformation acoustics

acoustic cloaking
Transformation acoustics

1. Slower
2. Matched impedance
Transformation acoustics

1. Slower
2. Matched impedance
   \[ \rho' c' = \rho c \]
   Denser
Transformation acoustics

1. Slower

2. Matched impedance

$$\rho' c' = \rho c$$

Denser

Same mass
Transformation acoustics

same for all incidence directions
Transformation acoustics

same for all incidence directions

speed depends on direction

“acoustic anisotropy”
\[ \frac{1}{K_s} = \frac{f}{K_0} + \frac{1-f}{K} \]

\[ \frac{1}{\rho_1} = \frac{f}{\rho_0} + \frac{1-f}{\rho} \]

\[ \rho_2 = f\rho_0 + (1-f)\rho \]

“homogenize”

acoustic anisotropy
Carpet cloak  aka  ground plane cloak
Acoustic carpet cloak

Zigoneanu et al. *Nature Mat.* 2014
Transformation acoustics

1D

2D

3D
Flexural wave cloaking

(a) without cloak  (b) with cloak

200 Hz  200 Hz

300 Hz  300 Hz

400 Hz  400 Hz

450 Hz  450 Hz

3 cm

collapsed

Stenger et al., *PRL* 2012
OUTLINE

Extraordinary acoustic transmission

Extraordinary acoustic absorption

Phononic crystals – some theory

Negative index materials

Transformation acoustics - cloaking
thanks:

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Office of Naval Research / MURI

and to you

for listening!
extra
Acoustic Poisson Effect (APE)

Elastic solid

energy scattered in perpendicular direction

N=2 shell flexural mode

constant volume mode

Acrylic shell, n=2 res. 15678Hz

$\rho_i$ 15.3kHz

Out of phase with incident wave

$\rho_s$ 16kHz

In phase with incident wave
Spacing and Coherence

- Acrylic shell in water
- $a=1\text{cm}$, $b=4.8\text{cm}$
- $n=2$ flexural mode
- Quadrupole-like standing wave in infinite array

IDEA: excite the array near the $n=2$ resonance with the spacing close to a wavelength

Scattered waves out of phase in the perpendicular direction

In phase
Elasticity

rigid cylinders

22172 Hz

elastic shells (APEL)

8 x 41 array (30cm x 142cm) of acrylic shells
Transient Gaussian Beam

APE

almost transparent

22172 Hz

19000 Hz
Guidelines for maximized interaction

- Select a shell with $n=2$ flexural resonance near $\lambda = 2b, \sqrt{2b}$
- Minimize distance from BG to flexural resonance
- Minimize shell spacing $b$