Symmetry and Reciprocity Breaking in Electromechanical Metamaterials and Structural Lattices

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Outline

• Overview

• Interface modes by breaking inversion symmetry
  o Concept: 1D spring mass lattice
  o 2D spring-mass hexagonal lattice
  o Experimental results: continuous hexagonal and Kagome lattices

• Time varying periodic systems
  o Time-periodic modulations
  o Time modulated systems
  o Preliminary experimental results

• Conclusions
Metamaterials: The quest for novel physical properties

- “Traditional” materials (new alloys, polymers): exploiting of chemical and material composition
- Metamaterials: exploiting of shape, symmetry, anisotropy and nonlinearity
  - Physical topology
  - Dispersion topology

Metamaterials:  
The quest for novel physical properties

Challenging classical notions in wave motion:  
reciprocity, sensitivity to defects (scattering), time-reversal symmetry
Topological Matter

- Topological insulators: a recent breakthrough in condensed matter physics:
  - One-way edge propagation determined by the electron spin
  - Insulators in bulk, conducting at edges
  - Immune to defects, no backscattering at edges
  - Energy bands with non-trivial topology
  - Surface states are eigenvectors of Hamiltonian operator

\[ ^1 \text{Nature} \ 464, \ 194-198 \ (\text{March, 2010}) \]
\[ ^2 \text{Nature Mat.s} \ 12, \ 233-239 \ (\text{March 2013}) \]
\[ ^3 \text{Nature} \ 496, \ 196-200 \ (11 \text{ April 2013}) \]
One-way edge propagation determined by the electron spin

Source

Yang et al., Topological Acoustics, PRL 114, 114301 (2015)
Boundary Modes in Mechanical/Acoustic Lattices


Geometric phase and band inversion in periodic acoustic systems

Meng Xiao$^{1,2}$, Guancong Ma$^{1,9}$, Zhiyu Yang$^1$, Ping Sheng$^{1,9}$, Z. Q. Zhang$^{1,2}$ and C. T. Chan$^{1,2}$

[Image of a setup with a loudspeaker and microphone, showing measurements]

[Graph showing frequency versus $\Delta d$ (cm)]

Valley Vortex States in Sonic Crystals

Jiuyang Lu, Chunyin Qiu, Manzhu Ke, and Zhengyou Liu

[Graphs illustrating valley vortex states with arrows indicating $\alpha = -10^\circ$, $\alpha = 0^\circ$, $\alpha = +10^\circ$]

\[ \text{sgn}(m) = \text{sgn}(\frac{\rho_{q^+}}{\rho_p}) \]

\[ m > 0 \]

\[ m < 0 \]

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Geometric symmetry-breaking for the generation of interface modes
Overview

• Concept provides the opportunity to obtain configurations of low geometrical complexity.

• The resulting structures could illustrate potential technological applications for wave guiding and isolation, and backscatter-robust transmission.

• Investigation of elastic (continuous) structures that emulate the Quantum Valley Hall effect (QVHE) to achieve topologically protected interface modes.


Spring-Mass Lattice

- Equations of motion
  
  \[
  \begin{align*}
  \left[-\omega^2 + (k_1 + k_2)\right]u_{1i} - k_2u_{2i} - k_1u_{1i-1} &= 0 \\
  \left[-\omega^2 + (k_1 + k_2)\right]u_{2i} - k_2u_{1i} - k_1u_{1i+1} &= 0
  \end{align*}
  \]

- Characteristic equation based on Bloch formalism

  \[
  \begin{bmatrix}
  \Omega^2 - 2 & (1 + \gamma) + (1 - \gamma)e^{-j\mu} \\
  (1 + \gamma) + (1 - \gamma)e^{j\mu} & \Omega^2 - 2
  \end{bmatrix}
  \begin{bmatrix}
  u_1 \\
  u_2
  \end{bmatrix} =
  \begin{bmatrix}
  0 \\
  0
  \end{bmatrix}
  \]

  \[
  \Omega^2 = 2 \pm \sqrt{2\left[1 + \gamma^2 + (1 - \gamma^2)\cos \mu\right]}^{1/2}
  \]
Dispersion relations

\[ \gamma > 0 \]

\[ \gamma < 0 \]
Band inversion
Wave modes and topological invariants

\( \gamma > 0 \)
Modes vs. propagation constant

\[ \gamma > 0 \]

Mode 1

\[ \mu = 0 \]

Mode 2

\[ \mu = \pi \]
Wave modes and topological invariants

$\gamma < 0$
Modes vs. propagation constant

\[ \gamma < 0 \]
Topological invariant: Zak Phase

\[ Z = \frac{i}{2\pi} \int_{-\pi}^{\pi} u^*(\kappa) \partial_\kappa u(\kappa) d\kappa \]

\[ Z = -\frac{i}{2\pi} \text{Im} \sum_{n=-N}^{N-1} \ln \left[ u^* \left( \frac{n}{N} \pi \right) \cdot u \left( \frac{n+\frac{1}{2}}{N} \pi \right) \right] \]

\( \gamma > 0 \)

\[ Z = 0 \]

\( \gamma < 0 \)

\[ Z = 1 \]


Interface modes
\( \gamma > 0 \) \hspace{2cm} \gamma < 0 \hspace{2cm} \Omega = \sqrt{3 - \sqrt{1 + 8\gamma^2}} \)
Localized mode

\[ \Omega = \sqrt{3 - \sqrt{1 + 8\gamma^2}} \]
\( \gamma > 0 \)

\[ \Omega = \sqrt{3 + \sqrt{1 + 8\gamma^2}} \]

\( \gamma < 0 \)

\[ \Omega = \sqrt{2} \]
Localized modes

\[ \Omega = \sqrt{2} \]

\[ \Omega = \sqrt{3 + \sqrt{1 + 8\gamma^2}} \]
Interface modes:
\[ \Omega = \sqrt{3 - \sqrt{1 + 8\gamma^2}} \]
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Interface modes and FRFs

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0 0.5 1 1.5 2

-40 -35 -30 -25 -20 -15

-10 -5

0 5

'H' Interface
**Governing equations**

\[
\begin{align*}
m_a \ddot{u}^a_{p,q} + k \left(3u^a_{p,q} - u^b_{p,q} + u^b_{p,q-1} - u^b_{p-1,q}\right) &= 0, \\
m_b \ddot{u}^b_{p,q} + k \left(3u^b_{p,q} - u^a_{p,q} + u^a_{p,q+1} - u^a_{p+1,q}\right) &= 0.
\end{align*}
\]

\[
m_a = (1 + \beta) m, \\
m_b = (1 - \beta) m
\]

**Plane wave solution**

\[
u_{p,q} = \nu_0 e^{i(\omega t + \kappa \cdot r_{p,q})}
\]

\[
\Omega^2 \begin{bmatrix} 1 + \beta & 0 \\ 0 & 1 - \beta \end{bmatrix} \begin{bmatrix} u^a \\ u^b \end{bmatrix} = \begin{bmatrix} 3 \\ -1 - e^{i\kappa \cdot a_1} - e^{i\kappa \cdot a_2} \end{bmatrix} \begin{bmatrix} -1 - e^{-i\kappa \cdot a_1} - e^{-i\kappa \cdot a_2} \\ 3 \end{bmatrix} \begin{bmatrix} u^a \\ u^b \end{bmatrix}.
\]

Band diagrams and topological invariants

\[ \beta = 0 \]

\[ \beta = 0.2 \]
Band inversion
Dispersion Analysis of a Finite Strip and Transient Simulations

unit cell

$\beta > 0$

$\beta < 0$

interface
Dispersion Analysis of a Finite Strip

‘L’ Interface

Ω

κ_x a/π

2.5
2
1.5
1
0.5
0
0.5
1
-0.5
-1

‘H’ Interface

Ω

κ_x a/π

2.5
2
1.5
1
0.5
0
0.5
1
-0.5
-1
Numerical simulations:
32 x 32 lattice with zig-zag ‘L’ interface
\[ \Omega_e = 1.5 \]
Numerical simulations:
32 x 32 lattice with zig-zag L interface
Thin plates with resonator arrays
Dirac Cones and Valley Modes

(*) D. Torrent, D. Mayou, J. Sanchez-dehesa
Elastic analog of graphene: dirac cones and edge states for flexural waves in thin plates
Physical Review B 87, 115143 (2013)
Governing equations

\[ D \nabla^4 w + \rho h \ddot{w} = k \sum_{\alpha} (w - w_\alpha) \delta (x - R_\alpha), \]

\[ m_\alpha \ddot{w}_\alpha + k (w_\alpha - w(R_\alpha)) = 0 \]

- Plane wave expansion method (PWEM)(*):
  \[ w(x, t) = e^{i\omega t + i\kappa \cdot x} \sum_G e^{ig \cdot x} W_G \]
  \[ w_\alpha(R_\alpha) = \sum_{\kappa} e^{i\kappa \cdot R_\alpha} W_\alpha(\kappa) \]

Plate with hexagonal array of resonators: dispersion
Waveguiding
Z-defect in plate array: trivial waveguide

$$\omega > \omega_{cut-off}$$
Observation of topological valley modes in an elastic hexagonal lattice
Configuration

Vila, Javier, Raj K. Pal, and Massimo Ruzzene
"Observation of topological valley modes in an elastic hexagonal lattice."
Phys. Rev. B 96, 134307
https://doi.org/10.1103/PhysRevB.96.134307
Scanning Laser Vibrometer Measurements
Experimental dispersion diagram investigation

\[ \hat{w}(\kappa_x, \kappa_y, \omega) = \mathcal{F}_{3D}[w(x, y, t)] \]

\[ |\hat{w}(\kappa|C, \omega)| \]

\[ |\hat{w}(\kappa_x, \kappa_y, \omega_0)| \]

\[ f_0 \approx 3.75 \, kHz \]
Experimental dispersion diagram investigation

\[ \gamma = 0 \]

\[ \gamma = 1 \]
Waveguiding along interfaces

\[ t_1 = 1.93 \text{ ms} \]

\[ t_2 = 3.89 \text{ ms} \]

\[ t_3 = 5.84 \times 10^{-8} \]

\[ t_1 = 1.54 \text{ ms} \]

\[ t_2 = 3.11 \text{ ms} \]

\[ t_3 = 4.67 \times 10^{-8} \]
Waveguiding along a linear interface
Waveguiding along interfaces
Trivial interface