ASME-NCAD
RAYLEIGH Lecture

COMPUTATIONAL VIBROACOUSTICS
in Low and Medium Frequency Bands

Roger Ohayon

Conservatoire National des Arts et Métiers (CNAM)
Structural Mechanics and Coupled Systems Laboratory, Paris, France

roger.ohayon@lecnam.net

IMECE, Pittsburgh, November 11-15, PA, USA, 2018
Acknowledgments

ASME-NCAD Committee (Dr. Shung H. Sung) for the honor of Rayleigh Lecture

Christian Soize, Professor (Emeritus), University of Paris-Est, MSME Lab. (Expert on UQ-Uncertainty Quantification and Computational Vibroacoustics/FSI in LF and MF ranges)

Jean-Sebastien Schotte, Research Scientist at Onera (computational hydroelasticity and sloshing) and Jean-Francois Deu, Professor at Cnam (smart structures)

Students and PhDs from whom I learned so much
OUTLINE

✓ LOW – LF and MEDIUM - MF FREQUENCY BANDS

✓ FSI / VIBROACOUSTICS (Structural Acoustics, Fluid Compressibility)

✓ FSI : HYDROELASTICITY-SLOSHING

✓ SMART SYSTEMS and FSI

Computational Vibration Analysis and Reduced Order Models
REFERENCES


Local references


AEROSPACE APPLICATIONS

- Engine vibrations
- Atmospheric turbulence
- Thrust fluctuations
- Attitude adjustment
- Equipment motion

By courtesy of Airbus Group

By courtesy of CNES

By courtesy of ESA
AEROSPACE APPLICATIONS

Liquid propelled launcher

Courtesy of Dassault Aviation
FREQUENCY DOMAIN

Measured Transfer Function

(OS-1998)
STRUCTURAL ACOUSTIC / FSI VIBRATIONS

Diagram showing mechanical and acoustical loadings interacting with a fluid and structure.
VIBROACOUSTIC / FSI SYSTEM

(OS-1998)
SYSTEM 1 + 2 (2 dissipative) equivalent to SYSTEM 1 + Interface frequency-dependent reaction forces

(OS-1998)
INTERIOR VIBROACOUSTIC – FSI SYSTEM
(OS-1998, 2014)

Using for instance Boundary Element Method (BEM)

\[ p_E|_{\Gamma_E}(\omega) = \left. p_{\text{given}} \right|_{\Gamma_E}(\omega) + i\omega Z_{\Gamma_E}(\omega) \{ u(\omega) \cdot n^S \} \]
Rayleigh Quotient and Vibration Modes

\[ R(u) = \frac{\int_{\Omega} \sigma_{ij}(u) \epsilon_{ij}(u) \, dx}{\int_{\Omega} \rho |u|^2 \, dx} \]

\[ Ku = \lambda Mu \]

Rayleigh or Proportional Damping

\[ D(\omega) = a_0(\omega) M + a_1(\omega) K \]

\[ (-\omega^2 M + i\omega D(\omega) + K(\omega)) u = f(\omega) \]
\(( -\omega^2 \mathbf{M} + i\omega \mathbf{D}(\omega) + \mathbf{K}(\omega) ) \mathbf{u} = \mathbf{f}(\omega) \)

\[ \mathbf{u}(\omega) = \mathbf{T}(\omega) \mathbf{f}(\omega) \]

\[ \mathbf{K}\mathbf{u} = \lambda \mathbf{M}\mathbf{u} \]
Low-frequency classical approach

For given forces

Free-free modes

\[ Ku = \lambda M u \]

\[ u = \sum_{\alpha=1}^{n} \frac{1}{-\omega^2 + \omega^2_{\alpha}} \int_{\partial \Omega} F \cdot u_{\alpha} d\sigma \frac{u_{\alpha}}{\mu_{\alpha}} \]
For prescribed accelerations (or velocities or displacements) (free free modes are replaced by constrained modes)
Medium-frequency range

- Increase of modal density with respect to frequency, which leads to huge models
- Inaccuracy to predict individual modal characteristics which reflects the various uncertainty sources inherent to modeling (in relation with mechanical or geometrical assumptions)
- Experimental identification of such models which become unreliable
Medium frequency methodologies

✓ Various methodologies in literature (see for instance, Nefske-Sung Power Flow Method, in which an equation of heat type – diffusion equation – replaces the usual elasticity equations, the unknown being an energy; Modal Sampling Method due to J.L. Guyader and Modal Hybrization Method due to H. Morand; Transfer Path Analysis (TPA), Energy Finite Element Analysis (EFEA), Wave-based methods (see HSN-2016)

✓ Let us describe a direct method developed by Christian Soize

\[
(-\omega^2 M + i\omega D(\omega) + K(\omega)) u = f(\omega)
\]

\[
u(\omega) = T(\omega) f(\omega)
\]
In a sub-band, the damping and the stiffness are supposed constant

\[
\mathcal{B}_\nu = [\Omega_\nu - \Delta\omega/2, \Omega_\nu + \Delta\omega/2]
\]

\[
\frac{\Delta\omega}{\Omega_\nu} \ll 1
\]

\[
\mathbf{f}(\omega) = \theta_\nu(\omega) \mathbf{b}
\]

\[
(-\omega^2 \mathbf{M} + i\omega \mathbf{D}(\omega) + \mathbf{K}(\omega)) \mathbf{u}(\omega) = \theta_\nu(\omega) \mathbf{b}
\]
In the time domain, shifting of MF to LF problem

\[
(-\omega^2 M + i\omega D_{\nu} + K_{\nu}) u_{\nu}(\omega) = \theta_{\nu}(\omega) b
\]

\[
u \equiv u_0(t) e^{i\Omega_{\nu} t}
\]

\[
M \partial_t^2 u_0(t) + \tilde{D}_{\nu} \partial_t u_0(t) + \tilde{K}_{\nu} u_0(t) = \theta_0(t) b
\]

\[
u = \frac{1}{B_{\nu}(\omega)} \Delta t \sum_{m \in \mathbb{Z}} u_0(m \Delta t) e^{-im \Delta t (\omega - \Omega_{\nu})}
\]
Medium-frequency range

- Inaccuracy to predict individual modal characteristics which reflects the various uncertainty sources inherent to modeling (in relation with mechanical or geometrical assumptions) BUT due to the new-computer area, a modal analysis can be carried on a much more efficient way AND the modal projection basis are very efficient to build REDUCED ORDER MODELS.
STRUCTURAL DYNAMICS OF COMPLEX STRUCTURE IN THE MF RANGE

PERIODIC STRUCTURAL SYSTEM

by courtesy of Onera
Accelerations Measurement and Computation

Energies Measurement and Computation

by courtesy of Onera (Soize's method)
REDUCED ORDER MODEL

Dynamic substructuring decomposition (Craig-Bampton-Hurty)

- Fixed/free eigenmodes
- Static interface deformations
Hybrid Degrees of Freedom Description

Interface physical degrees of freedom – Internal Generalized Coordinates

\[
\begin{bmatrix}
K^s & \cdots & 0 & \cdots \\
\vdots & & \ddots & \\
0 & \cdots & \omega^2_{\alpha} & \mu_{\alpha} \\
\vdots & & \cdots & \\
\end{bmatrix} \begin{bmatrix} U_{\Sigma} \end{bmatrix} - \omega^2 \begin{bmatrix}
M^s & \cdots & C_{\alpha} & \cdots \\
\vdots & & \ddots & \\
C^{T}_{\alpha} & \cdots & \mu_{\alpha} \\
\vdots & \cdots & \cdots & \\
\end{bmatrix} \begin{bmatrix} U_{\Sigma} \end{bmatrix} = \begin{bmatrix} F_{\Sigma} \end{bmatrix}.
\]
COUPLING OF SUBSTRUCTURES
FOR VIBRATION ANALYSIS IN THE MF RANGE

Generic structure

by courtesy of Onera
Convergence Aspects

by courtesy of Onera, Soize’s method
INTERIOR VIBROACOUSTIC – FSI SYSTEM
(OS-1998, 2014)

Using for instance Boundary Element Method (BEM)

\[ p_E|_{\Gamma_E}(\omega) = p_{\text{given}}|_{\Gamma_E}(\omega) + i\omega Z_{\Gamma_E}(\omega) \{ u(\omega) \cdot n^S \} \]
Non-homogeneous heavy compressible fluid

Dynamic of liquids

- Incompressible homogeneous sloshing
- Compressible homogeneous acoustic
- Compressible non-homogeneous internal gravity waves

Plane irrotationality
Hydroelastic model with gravity

Sloshing  Hydroelasticity  Vibroacoustics

Liquid

Structure

hydroelasticity with gravity  and compressibility

compressible
FSI Vibrations
Some Non-exhaustive Basic Works

✓ W.C. Fung, P. Tong
✓ N. Abramson, F. Dodge, F. Kana, Housner
✓ O.C. Zienkiewicz, Newton
✓ T. Belytschko, T. Hughes, W.K. Liu
✓ T. Geers, C. Felippa, K.C. Park,
✓ Fraeijs de Veubeke, M. Géradin, C. Farhat
✓ Ibrahim, Faltinsen, Paidoussis, Tezduyar, Takizawa, Bazilevs
STRUCTURAL-ACOUSTIC EQUATIONS
Structure submitted to a fluid pressure loading

\[ KU - \omega^2 MU - Cp = F \]

- structural stiffness
- structural mass
- fluid-structure coupling

\[ K = K_E + K_G + K_{P_0} \]

- elastic stiffness
- geometric stiffness
- follower forces
LOCAL FLUID EQUATIONS IN TERMS OF PRESSURE AND WALL NORMAL DISPLACEMENT

Helmholtz equation

$$\nabla^2 p + \frac{\omega^2}{c^2} p = 0 \quad \text{in} \quad \Omega_F$$

Kinematic boundary condition

$$\frac{\partial p}{\partial n} = \rho_F \omega^2 u.n \quad \text{on} \quad \Sigma$$

with the constraint

$$\frac{1}{\rho_F c^2} \int_{\Omega_F} p \, dx + \int_{\Sigma} u.n \, d\sigma = 0$$
Internal Fluid Damping

\[-\frac{\omega^2}{\rho_0 c_0^2} p - i\omega \frac{\tau}{\rho_0} \nabla^2 p - \frac{1}{\rho_0} \nabla^2 p = \frac{1}{\rho_0} (i\omega Q - \tau c_0^2 \nabla^2 Q)\]

Cf A. Pierce (with a complex wave number)
Finite Element Discretization and Matrix Equation

\[
\begin{bmatrix}
    A_{\text{FSI}}(\omega) \\
    U(\omega) \\
    P(\omega)
\end{bmatrix}
= 
\begin{bmatrix}
    F^S(\omega) \\
    F(\omega)
\end{bmatrix}
\]
\[
\left[ A_{\text{FSI}}(\omega) \right] = \begin{bmatrix}
\left[ A^S(\omega) \right] - \omega^2 \left[ A_{\text{BEM}}(\omega/c_E) \right] & [C] \\
\omega^2 [C]^T & \left[ A(\omega) \right] + \left[ A^Z(\omega) \right]
\end{bmatrix}
\]
\[
[A^S(\omega)] = -\omega^2[M^S] + i\omega [D^S(\omega)] + [K^S(\omega)]
\]

\[
[A(\omega)] = -\omega^2[M] + i\omega [D] + [K]
\]

\[
[D] = \tau [K]
\]
REDUCED ORDER MODEL
Direct Modal Analysis vs Dynamic Component Mode
(substructuring approach)

\[
\begin{bmatrix}
K & 0 & 0 \\
0 & K_p & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U \\
p \\
\phi_1
\end{bmatrix}
= \lambda
\begin{bmatrix}
M + A_2 F_{22}^{-1} A_2^T & A_2 F_{22}^{-1} B_2^T & a \\
B_2 F_{22}^{-1} A_2^T & B_2 F_{22}^{-1} B_2^T & b \\
 a^T & b^T & 0
\end{bmatrix}
\begin{bmatrix}
U \\
p \\
\phi_1
\end{bmatrix}
\]

which exhibits the matrix \( M_A^{(u,p,\phi_1)} \), called the added mass matrix, which is expressed:

\[
M_A^{(u,p,\phi_1)} =
\begin{bmatrix}
A_2 F_{22}^{-1} A_2^T & A_2 F_{22}^{-1} B_2^T & a \\
B_2 F_{22}^{-1} A_2^T & B_2 F_{22}^{-1} B_2^T & b \\
 a^T & b^T & 0
\end{bmatrix}
\]

Unpracticable !!!
STRUCTURAL-ACOUSTIC REDUCED ORDER MODEL

Dynamic substructuring decomposition (Craig-Bampton-Hurty)

- Fixed/free eigenmodes
- Static interface deformations
PHYSICAL DECOMPOSITION
acoustic puzzle pieces

Solution searched under the following form:

\[ p = p^s (u.n) + \sum_{\alpha=1}^{N_p} r_{\alpha} p_{\alpha} \]

\( r_{\alpha} \) generalized acoustic unknowns

Quasi-static response \( p^s \)

Acoustic modes \( p_{\alpha} \)
Acoustic modes in a rigid motionless cavity

\[ K_P p_\alpha = \omega_{\alpha}^2 F p_\alpha \]

with orthogonality conditions

Quasi-static pressure response

\[ p^s = -\frac{\rho_F c^2}{|\Omega_F|} \int_{\Sigma} u.n \, d\sigma \]
Structural-Acoustic Reduced Order Model

Hybrid FE/generalized coordinates representation

\[
\begin{pmatrix}
K^{\text{tot}} & 0 \\
0 & \text{Diag} \left( \mu_\alpha \right)
\end{pmatrix}
\begin{pmatrix}
U \\
r
\end{pmatrix}
- \omega^2
\begin{pmatrix}
M^{\text{tot}} & D \\
D^T & \text{Diag} \left( \mu_\alpha / \omega^2_\alpha \right)
\end{pmatrix}
\begin{pmatrix}
U \\
r
\end{pmatrix}
= \begin{pmatrix}
F^d \\
0
\end{pmatrix}
\]

with

\[
K^{\text{tot}} = \tilde{K} + K^s
\]

\[
M^{\text{tot}} = M + \sum_{\alpha=1}^{N_p} \frac{1}{\omega^2_\alpha \mu_\alpha} C_\alpha C^T_\alpha
\]

\[
D_\alpha = \sum_{\alpha=1}^{N_p} \frac{1}{\omega^2_\alpha} C_\alpha
\]

← gas pneumatic effect
Passive treatments

**Porous materials** (foams, glass wools...) mounted on the acoustic cavity walls
- Acoustic absorption

Active treatments

➡️ **Intelligent materials** (piezoelectric sensors and actuators) attached to the vibrating structures
- Energy dissipation and vibration control (electrical circuits, controllers)
Noise and vibration reduction in internal vibroacoustic problems

Absorbing interface

Adaptive structures
Interface wall damping impedance effects

\[ \Delta p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad \text{in } \Omega_F \]

\[ \nabla p \cdot n^F = 0 \quad \text{on } \Gamma_R \]

\[ \nabla p \cdot n^F = -\rho_F \frac{\partial^2 \eta}{\partial t^2} \quad \text{on } \Gamma_A \]

\[ p = k^I \eta + d^I \frac{\partial \eta}{\partial t} \quad \text{on } \Gamma_A \]
✓ Inviscid, compressible, barotropic fluid
✓ Piezoelectric structure with linear constitutive equations:
**General problem and hypotheses**

Elastic structure equipped with $P$ piezo patches and filled with fluid

- Thin piezo patches with constant thickness $h^{(p)}$
- Transverse polarization (normal to the electrodes)
- Thickness of the electrodes neglected
- No inter piezo coupling
- Electric displacement neglected in the elastic domain $\Omega_s$

Electric field normal to the electrodes and uniform in the $p$-th patch:

$$E_k = -\frac{\psi^{(p)} - \psi^{(p)}}{h^{(p)}} n_k = \frac{V^{(p)}}{h^{(p)}} n_k$$

Only a couple of variables by piezoelectric patch:

Potential difference $V^{(p)}$ and free electric charge $Q^{(p)}$ [Thomas, Deü 2009 & al]
PIEZOELECTRIC CONSTITUTIVE EQUATIONS

in terms of
structural displacement and electric potential fields \((u, \psi)\)

\[
\begin{align*}
\sigma_{ij} &= c_{ijkl} \varepsilon_{kl}(u) - e_{kij} E_k(\psi) \\
D_i &= e_{ikl} \varepsilon_{kl}(u) + \varepsilon_{ik} E_k(\psi)
\end{align*}
\]

with

\[
\begin{align*}
\varepsilon_{kl} &= \frac{1}{2} (u_{k,l} + u_{l,k}) \\
E_k &= -\psi_{,k}
\end{align*}
\]

E: electric field vector
D: electric displacement field vector

e: piezoelectric 3\textsuperscript{rd} order tensor
\(\varepsilon\): dielectric 2\textsuperscript{nd} order tensor
PIEZOELECTRIC STRUCTURAL-ACOUSTIC PROBLEM
Local Equations in terms of \((u, \psi, p)\)

Piezoelectric structure

\[
\begin{align*}
\sigma_{ij,j} + \omega^2 \rho_S u_i &= 0 \quad \text{in } \Omega_S \\
\sigma_{ij} n_j^S &= 0 \quad \text{on } \Gamma_\sigma \\
u_i &= 0 \quad \text{on } \Gamma_u \\
\sigma_{ij} n_j^S &= p n_i \quad \text{on } \Sigma
\end{align*}
\]

where

\[
\begin{align*}
D_{i,i} &= 0 \quad \text{in } \Omega_S \\
D_i n_i^S &= 0 \quad \text{on } \Gamma_D \\
\psi &= 0 \quad \text{on } \Gamma_\psi
\end{align*}
\]

Acoustic fluid

\[
\begin{align*}
p_{,ii} + \frac{\omega^2}{c_F^2} p &= 0 \quad \text{in } \Omega_F \\
p_{,i} n_i &= \omega^2 \rho_F u_i n_i \quad \text{on } \Sigma
\end{align*}
\]
PIEZOELECTRIC STRUCTURAL-ACOUSTIC PROBLEM
Finite element discretization

\[
\begin{pmatrix}
K_u & -C_{up} & -C_{up} \\
C_{up}^T & K_\psi & 0 \\
0 & 0 & K_p
\end{pmatrix}
\begin{pmatrix}
U \\
\Psi \\
P
\end{pmatrix}
- \omega^2
\begin{pmatrix}
M_u & 0 & 0 \\
0 & 0 & 0 \\
C_{up}^T & 0 & M_p
\end{pmatrix}
\begin{pmatrix}
U \\
\Psi \\
P
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]
Remarks:

✓ Elimination of electric potential degrees of freedom leads to

\[
\begin{pmatrix}
K_u + K^A & -C_{up} \\
0 & K_p
\end{pmatrix}
\begin{pmatrix}
U \\
P
\end{pmatrix}
- \omega^2
\begin{pmatrix}
M_u & 0 \\
C_u^T & M_p
\end{pmatrix}
\begin{pmatrix}
U \\
P
\end{pmatrix}
=
\begin{pmatrix}
0 \\
0
\end{pmatrix}
\]

where we introduce the piezoelectric added-stiffness operator

\[
K^A = C_{u\psi}K_{\psi}^{-1}C_u^{T\psi}
\]

is due to the electromechanical coupling

✓ Symmetric formulations can be established by introducing a fluid displacement potential as additional variable
FLUID-STRUCTURE REDUCED ORDER MODEL

Quasi-static response $p^s$

Acoustic modes $p_\alpha$

Smart system

Structure modes $U_\beta$

Generalized coordinates representation

$U = \sum_{\beta=1}^{N_u} q_\beta U_\beta$

with

$K^{\text{tot}} U_\beta = \lambda_\beta M^{\text{tot}} U_\beta$

$(U,V,P) \quad \Rightarrow \quad (U,V,r)$

$(U,V,r) \quad \Rightarrow \quad (q,V,r)$
Structural-acoustics with dissipative interface
2D example

Absorbing wall

\[ k^I = 5 \times 10^6 \text{ N/m}^3 \text{ and } d^I = 50 \text{ N.s/m}^3 \]

Air

\[ \rho = 1 \text{ kg.m}^{-3} \]
\[ c = 340 \text{ m.s}^{-1} \]

\[ Z(\omega) = d^I + i \frac{k^I}{\omega} \]
Structural-acoustics with dissipative interface
3D example
Fluid response

- the peaks correspond to the computed natural frequencies
- only the fluid modes are influenced by the interface
**RL resonant shunt**

Structural acoustic problem with one piezoelectric patch connected to RL series shunt circuit

\[ L\ddot{Q} + R\dot{Q} + V = 0 \]

\( \Rightarrow \) Optimal values of \( R \) and \( L \) in order to maximize the attenuation of one particular mode
Active Twist Rotor Blades for Helicopter

Manageable Adaptive Twist Rotor for Improved Control and Speed

- **Aim of Project**: Decrease rotor vibration and noise by adaptation of blade twist angle vs flight phase.

- **Definition of Rotor Blades**: Based on TWISCA (TWIstable Section Closed by Actuator) Onera patent: open carbon fiber blade with 40 MFC piezoelectric actuators distributed along the span.

- **Objective performance**: $\pm$ 2° @ 2500V

- **5 Blades available – Several defects on actuators**

<table>
<thead>
<tr>
<th>Blade Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuators efficiency</td>
<td>97.35%</td>
<td>87.46%</td>
<td>70%</td>
<td>87.50%</td>
<td>87.20%</td>
</tr>
<tr>
<td>Number of broken actuators</td>
<td>1</td>
<td>5</td>
<td>12</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Input Voltage</td>
<td>2380V</td>
<td>2400V</td>
<td>2376V</td>
<td>2375V</td>
<td>2383V</td>
</tr>
<tr>
<td>Torsion Angle</td>
<td>1.81°</td>
<td>1.79°</td>
<td>1.26°</td>
<td>1.49°</td>
<td>1.41°</td>
</tr>
<tr>
<td>Torsion Angle at 2500V</td>
<td>1.90°</td>
<td>1.88°</td>
<td>1.33</td>
<td>1.57</td>
<td>1.50°</td>
</tr>
</tbody>
</table>

Evolution of blade twist angle vs input voltage at 2 different radius

by courtesy of Onera
Noise reduction in helicopter cabin with an optimized acoustic composite panel

by courtesy of Onera
Noise reduction in helicopter cabin with an optimized acoustic composite panel

Panel test with acoustic excitations

Sum on 8 accelerometers (Reduction on RMS level = 2.87 dB)
Noise reduction in helicopter cabin with an optimized acoustic composite panel

Composite Helicopter cabin with mechanical and acoustic excitations

by courtesy of ONERA
Noise reduction in helicopter cabin with an optimized acoustic composite panel

Ground and flight test

Large reduction of vibration level
Minor effect on internal noise

by courtesy of Onera
Vibration reduction at launcher / payload interface

Problem
Large forces and displacements

Magneto-Rheological (MR) Fluid Technology

Principle
The application of a magnetic field increases the viscosity of MR fluid, therefore the damping

by courtesy of Onera
HYDROELASTIC-SLOSHING VIBRATIONS

EXPERIMENTAL CONSIDERATIONS

acoustic domain

Frequency

Hz

free surface sloshing
hydroelastic deformation
STRUCTURAL ACOUSTIC EQUATIONS
Structure submitted to a fluid pressure loading

\[ KU - \omega^2 MU - Cp = F \]

- structural stiffness
- structural mass
- fluid-structure coupling

\[ K = K_E + K_G + K_g \]

- elastic stiffness
- Geometric stiffness
- Follower forces

\[ \Omega^F \]
\[ \Omega^s \]
\[ \Gamma \]
Local liquid equations in terms of pressure and wall normal displacement

Incompressibility assumption
\[ \nabla^2 p = 0 \]
in \[ \Omega_F \]

Kinematic boundary condition
\[ \frac{\partial p}{\partial n} = \rho_F \omega^2 u.n \]
on \[ \Sigma \]

Free surface condition
\[ \frac{\partial p}{\partial z} = \frac{\omega^2}{g} p \]
on \[ \Gamma \]

with the constraint
\[ \frac{1}{\rho_F g} \int_{\Gamma} p \, d\sigma + \int_{\Sigma} u.n \, d\sigma = 0 \]
REDUCED ORDER MODEL

Dynamic physical decomposition

- sloshing modes in rigid motionless cavity
- quasi-static pressure response to normal structure deformation

\[ u_N = 0 \]
PHYSICAL DECOMPOSITION
liquid puzzle pieces

Solution searched under the following form:

\[ p = p^s(u\cdot n) + \sum_{\alpha=1}^{N_p} r_\alpha p_\alpha \]

\( r_\alpha \) generalized sloshing unknowns

Quasi-static response \( p^s \)

Sloshing modes \( p_\alpha \)
Sloshing modes in a rigid motionless cavity

Sloshing modes \( p_\alpha \)

Quasi-static pressure response \( p^s \)

\[
K_\Gamma p_\alpha = \omega_\alpha^2 F p_\alpha
\]

with orthogonality conditions

\[
p^s = -\frac{\rho F g}{|\Gamma|} \int_\Sigma u.n \, d\sigma
\]
LIQUID-STRUCTURE REDUCED ORDER MODEL

Hybrid FE/generalized coordinates representation

\[
\begin{pmatrix}
K^{\text{tot}} & 0 \\
0 & \text{Diag } \mu_{\alpha}
\end{pmatrix}
\begin{pmatrix}
U \\
r
\end{pmatrix}
- \omega^2
\begin{pmatrix}
M^{\text{tot}} & D \\
D^T & \text{Diag}(\mu_{\alpha}/\omega^2_{\alpha})
\end{pmatrix}
\begin{pmatrix}
U \\
r
\end{pmatrix}
=
\begin{pmatrix}
F^d \\
0
\end{pmatrix}
\]
POSITION OF TAIL WING TANKS

Courtesy of Airbus Group
EFFECT OF FUEL SLOSHING ON FLUTTER

Farhat, Chiu, Schotte, RO (2014)
Collaboration Stanford – Cnam/LMSSC - Onera
Liquid-structure interaction in microgravity

Sloshing modes with capillarity $p_\alpha$

Free surface condition

- without surface tension:
  \[ p = \rho_F g \eta \]

- with surface tension:
  \[ p = \rho_F g (i_z \cdot \mathbf{n}) \eta + \mathcal{L}(\eta) \]
  on $\Gamma$

with
\[ \mathcal{L}(\eta) = -\sigma \left[ \left( \frac{1}{R_1^2} + \frac{1}{R_2^2} \right) \eta + \text{div}_S \nabla_S \eta \right] \]

with a contact angle boundary condition
Static equilibrium position of the liquid
Example: Spherical tank

Utsumi (1998)
HYDROELASTICITY / SLOSHING
Experimental validation

Collaboration with Professor Marco Amabili (Mc Gill) and Onera
ADAPTIVE FLUID-STRUCTURE SYSTEM (PZT)

Electrodynamical shaker exciting the plate through a thin stinger

- Natural frequencies
- Damping ratios
- Modal shapes

Comparison of the results before and after gluing the control components

Experimental set-up for modal analysis; tank partially filled with water.
ADAPTIVE FLUID-STRUCTURE SYSTEM (PZT)

- Tank equipped with control components
- Numerical simulations of the deformation energy of the plate
- Accelerometers and PZT patches glued on the plate
- Optimization of the positioning of the PZT patches
ADAPTIVE FLUID-STRUCTURE SYSTEM
White Noise Excitation (0-400Hz)
Water-Filled Tank

M. Amabili (McGill) and Onera collaboration

Measured FRF from accelerometer 1 at high frequencies: — no control, — SISO control, — MultiSISO control.
SLOSHING MODAL DENSITY

sloshing modal density

potential energy of sloshing modes
Damping sources for liquid sloshing

Henderson & Miles (1994), Wang et al. (2006)

"damping by wetting" at the contact line

viscous damping at the structural wall

viscous dissipation at the free surface

viscous damping in the interior fluid
CONCLUSION – OPEN PROBLEMS

✓ Dissipative Adaptive Interface Modeling

✓ Appropriate Reduced Order Models for Broadband Frequency Domains

✓ Hybrid Passive / Active Treatments for Vibrations and Noise Reduction

✓ Hybrid Passive / Active Treatments for Hydroelastic /Sloshing

✓ Nonlinearities (Vibrations / Transient Impacts and Shocks)
Acknowledgments

ASME-NCAD Committee (Dr. Shung H. Sung) for the honor of Rayleigh Lecture

Christian Soize, Professor (Emeritus), University of Paris-Est, MSME Lab. (Expert on UQ-Uncertainty Quantification and Computational Vibroacoustics/FSI in LF and MF ranges)

Jean-Sebastien Schotte, Research Scientist at Onera (computational hydroelasticity and sloshing) and Onera is acknowledged for the various applications presented

Students and PhDs from whom I learned so much
Thank you for your attention